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A TENTATIVE LIST OF OBJECTIVES IN THE TEACHING OF JUNIOR HIGH SCHOOL MATHEMATICS WITH INVESTIGATIONS FOR THE DETERMINING OF THEIR VALIDITY

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PREFACE

This study attempted to do five things: (1) to review and summarize the studies made by other workers in the field; (2) to advance the line by reports on special problems bearing on the general topic; (3) to construct a list of specific objectives using five criteria; (4) to present a more complete picture of the strength of mental connections in arithmetic for pupils at the beginning of the seventh grade; and (5) to formulate a technique whereby materials written for children may be adjusted on the basis of classroom trial under widely diversified conditions.

Certain changes are taking place in the teaching of mathematics in grades seven, eight, and nine. It is quite important that the new courses be definite in their aims. The present study includes 454 specific elements under the classification: *attitudes, concepts, abiilties,* and *items of information.* Of these 305 are in the basic list. This basic list was the result of reducing a tentative list of 519 objectives. It is possible that if the author had started with twice as many items in the original list, that a few more elements would have been added to the basic list. The chances are good, however, that very little that is fundamental in mathematics for these three grades has been omitted. There will be revision of this basic list and there will be advances in the way of making the items even more definite, but there probably will be few changes in the choice of the basic fundamental materials.

It appears that we have advanced a little in education when we can list our outcomes specifically instead of asserting that we teach a year, or two years, or three years of mathematics. No definite list of outcomes as yet is available in social studies, science or even first year Latin. The reader will be interested to examine the bases for the selection of materials as here presented.

The author is indebted to Dr. Otis W. Caldwell for generous financial support during the years in which the classroom investigations were carried forward. But for his constant encouragement support, and keen intuition, the work would not have been possible. During the six years that the writer

worked with Mr. Caldwell at the Lincoln School, the central theme of this study consumed the major part of his energy and thinking.

The author is most deeply obligated to Dr. John R. Clark for the stimulus, enthusiasm, and intelligence which he contributed during the three years of co-operative efforts on this problem at the Lincoln School.

The author wishes to express his gratitude to Professors T. H. Briggs and C. B. Upton for their valuable guidance in the definition of the problem herein reported. He wishes also to acknowledge with sincere thanks the co-operation of his mathematics jury,—Mr. Charles M. Austin, Mr. William Betz, Dean John H. Minnick, Professor Clifford B. Upton, Dr. John R. Clark, and the jury of general educators,—Professors Thomas H. Briggs, Stuart A. Courtis, Calvin O. Davis, Clifford Woody, and Dr. Charles L. Spain. Theirs was the arduous task of giving a judgment on each of more than five hunded elements.

The author believes that it is of the utmost importance that someone in each field bring together the evidence that should guide our practice, he is hopeful that the following pages will do this for the mathematics of grades seven, eight, and nine.

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A TENTATIVE LIST OF OBJECTIVES IN THE TEACHING OF JUNIOR HIGH SCHOOL MATHEMATICS WITH INVESTIGATIONS FOR THE DETERMINING OF THEIR VALIDITY

Introduction

"The Present Status of Mathematics in Grades Seven, Eight and Nine—There appears to be widespread dissatisfaction with the content and the organization of traditional materials of instruction in these grades. In particular, the work of these years is characterized by waste of time and effort in the teaching of insignificant subject matter. There is little defense against the charges that the conventional seventh and eighth grade mathematics devotes much time to unmotivated and unproductive reviews; that much of the social-economic arithmetic (taxes, insurance, stocks, bonds, and the like) is beyond the comprehension of the pupils of these earlier grades; that common sense estimation in solving problems does not receive adequate exercise; and that American practice has not begun to profit by the experience of European schools in giving to pupils of these grades an outlook and an appreciation of the significance and meaning of mathematics beyond arithmetic.

"Even less defense is offered for the conventional algebra of the ninth school year. Conservative mathematicians have recognized the need of reconstruction in this grade. Extreme emphasis upon the manipulation of symbolism, which to the pupil does not have meaning and significance, together with poorly selected content and material of little value even in later courses of mathematics, constitute the two pronounced indictments of the standard algebra of the ninth grade. Progressive teachers of mathematics and many school administrators are demanding a reconsideration of the content and methods of teaching in this grade."

The preceding paragraphs were written at the time* that

*The Mathematics Teacher, Vol. XIV, No. 5, Page 264. An Article by Raleigh Schorling and John R. Clark.

the investigations, to be presented in this book, were undertaken. The indictment is severe. What evidence is there that this statement is not that of extremists or destructionists who perhaps have taken their "reorganization courses in Education" too seriously? For an answer let us turn to a memorandum written at about the same time and addressed to the General Education Board by a committee appointed by the Mathematical Association of America. This memorandum resulted in the securing of generous funds for the National Committee on Mathematical Requirements. It enabled the Committee to prepare and publish its final report entitled, "The Reorganization of Mathematics in Secondary Education." Hence, the document will probably be worthy of note for the student of the history of American education. The memorandum bears on this discussion because it includes what are probably the severest and most startling criticisms ever made of the teaching of mathematics in our secondary schools. A few of the most significant statements from that memorandum are given here.

"The situation that needs to be met may best be illustrated by the case of algebra. Our elementary algebra is, in theory and symbolism, substantially what it was in the seventeenth century. The present standards of drill work, largely on non-essentials, were set up about fifty years ago. The few lines of application of algebra to nominally practical questions found in our college entrance examinations are mere variants of problems that are centuries old, and that often represent only remotely any real conditions of today. A considerable number of teachers, both in the Secondary Schools and the colleges, believe that the amount of time spent by pupils on abstract work in difficult problems in division, factoring, fractions, simultaneous equations, radicals, etc., is excessive; that such work leads to nothing important in the science, and adds but little to facility in the manipulation of algebraic forms."

"It is urged by many friendly critics that instead of giving the students a good all-round idea of what mathematics means and its general range of application our present secondary school courses are too abstract, often uninteresting, except to the mathematically inclined, and not as valuable as they might be as an aid to college in general, or to life work for those who enter at once into their careers."

Bear in mind that the statement is signed by all who were members of the Committee at the time the document was issued and that all members were teachers of mathematics, representing officially the large mathematical organizations—collegiate and secondary—of this country. No further evidence is necessary; the case against mathematics was frankly admitted by those presumably competent to judge.

A Program for Improvement. What practical steps could be taken to increase the efficiency of mathematics instruction in grades seven, eight and nine? This was precisely the problem that was delegated, with the promise of generous financial support and adequate study facilities, to the writer by the authorities* of the Lincoln School in the Spring of 1917.

THE PROBLEM

The Determination of Specific Objectives for Mathematics in Grades Seven, Eight and Nine. In dealing with a special field in the high school, as for example that of mathematics, there are three fundamental problems:

(1) What pupils shall be permitted to elect the subject?
(2) What materials shall be selected?
(3) How shall these materials be taught?

The crucial question in the present study is the *selection of materials for mathematics in the junior high school grades on the basis of five criteria.* The writer is not willing to admit that the question can be considered independent of the philosophy of education. Moreover it is necessary to consider what happens when these materials are taken into the classroom. Hence, it is important to keep in mind throughout the study the outline which follows:

THE GENERAL PROCEDURE

I. The formulation of a brief list of guiding principles implying a philosophy of secondary education.
II. The selection of specific objectives on the basis of the following criteria:
   (a) A summary of the elements for which some kind of a positive case can now be made by employing one or more objective studies dealing with the social uses of mathematics.
   (b) Practice as determined by an inventory of selected courses of study.
      (1) For seventh and eighth grades, certain pages quoted from Guiler's thesis. For ninth grade, a similar study by Schorling.
   (c) The outline of topics for the junior high school as given in certain pages in the report of the National Committee on Mathematical Requirements.

*See statement by Dr. Otis W. Caldwell in the Introduction to "Mathematics for the Eighth School Year, Schorling-Clark," World Book Company, Yonkers, New York.

(d) An inventory of the contents of the first seven series of mathematical texts written for the junior high school grades.

(e) A highly selected jury of five educators especially interested in the junior high school and five leaders in the teaching of high school mathematics.

III. The refinement of the elements selected and their grade placement by means of an extensive classroom trial under *test* and *report* conditions.

As sidelights on the general problem, certain subsidiary studies have been made and will be reported in the following pages. The data involving classroom investigations were collected by Dr. John R. Clark and the writer during their three years' investigation of learning problems in a considerable number of class rooms. The following is a list of studies that proved to be helpful:

(1) To administer an inventory test designed to reveal what the first six years of arithmetic achieves with reference to certain well-recognized objectives. This material, collected by the inventory, to be supplemented by results secured through widely used existing tests.

(2) To measure the progress of practice by the study of the content of an arithmetic very widely used twenty years ago in contrast with the content of an arithmetic having extensive sales today, together with a similar study of two ninth grade algebras. (To be published early in an article in the Mathematics Teacher.

(3) To make an analysis of the content of seven series of junior high school books to discover general tendencies in the different offerings of those groups that are presenting new materials to children in these grades.

(4) To make an analysis of materials used in schools undertaking innovations (so-called experiments) to discover significant "leads" from the work of unusually well-prepared teachers who are given almost complete freedom from traditional demands. (Published as Chapter XII in The Reorganization of Mathematics in Secondary Schools.)

(5) To survey the objective studies that throw some light on the selection of elements constituting the subject matter of these grades and to supplement these studies in two important sectors of the field.

(6) To list a specific set of pupil outcomes for the three junior high school grades considered as a unit. (Chapter V.)

(7) To search the literature of the philosophy of secondary education, of educational psychology, and of committee reports dealing with the mathematics of the secondary school as a basis for the formulation of a set of guiding principles which would assist in the selection of subject matter and which would control the development of text materials. (Chapter I.)

(8) To cooperate in a writing-testing-teaching-revision program with the hope of refining the list of objectives for these grades and to measure the effectiveness of the program by a series of tests which secure results on one of the most important issues,—the improvement of computation with whole numbers, common fractions, decimals, per cent, and common units of measure.

It is the purpose of this report to describe the steps of our program and present the results of our three years' study. There will be a chapter on each of the major steps, but before we develop these units it will be helpful to give a preliminary survey statement of each of the chapters.

A. *To Administer an Inventory Test Designed to Reveal What the First Six Years of Arithmetic Achieves with Reference to Certain Well-Recognized Objectives. This Test Material to be Supplemented by Results Secured Through Widely Used Existing Tests.*

A test of 125 elements was constructed (the criteria and method will be discussed later) and administered to a considerable number of beginning seventh grade pupils of different racial and social groups in thirty-one cities. The test was practically an untimed test,—almost all the pupils attempted all of these 125 elements. Probably Thorndike would state the purpose of this test to be the discovery of the strength of arithmetical connections. At any rate, he has since made and reported on a similar study in the field of algebra which attempts to show the degree of mastery for twenty-eight elements. The existing tests showed fairly well the ability of children to compute with integers and fractions and perhaps told us something about their ability to reason. The new test furnished information concerning the knowledge of symbols, processes, terms, principles and relations which beginning seventh grade children may be expected to possess.

The remainder of the inventory consists of a brief compilation from existing standardized test results, sufficient to give a reasonably complete picture of the degree of mastery of arithmetical principles and processes achieved in typical school conditions. The giving of this inventory test had probably a most wholesome and steadying effect on a group of men undertaking a new organization of materials. The combined results form the basis for the diagnosis and the remedial treatment reported in a later chapter.

This chapter reveals then a reasonably complete picture of the degree of mastery of arithmetic practices and processes achieved in typical school conditions. It shows that the connections are so weak that a conscious effort must be made to remedy the situation. Definite suggestions for improved methods of learning are made and one of these suggestions is checked by means of a widespread study in actual classroom situations. The statistical table for this check is presented in Chapter VI. The chief outcome of this chapter, however, is the clear evidence that *we need a list of specific objectives.*

B. *To Make an Analysis of the Content of Seven Series of Junior High School Books to Discover General Tendencies in the Different Offerings of Those Groups That Are Presenting New Materials to Children in These Grades.*

It should be said at the beginning that the content of the existing series of texts is not the sole valid criterion for the selection of subject matter. Nevertheless this study presented a bird's-eye view of the tendencies in the groups that were setting up programs to improve mathematics instruction. The study presented clear evidence of the fact that the mathematics problems of these grades had not been satisfactorily solved by revealing the marked variation shown in content and grade placement of the various books. We have clear evidence of distinctly different hypotheses concerning the curricula for these grades. One writer, for example, holds that the seventh grade should be given entirely to arithmetic and the eighth and ninth grades to general mathematics. Another group presents the general mathematics in grades seven and eight, and confines the ninth grade almost wholly to algebra. Still another group advocates geometry as the major work of the seventh grade, arithmetic for the eighth, and algebra for the ninth. A fourth series of books presents a different unit for each half year. For example, the first half of the seventh grade is all arith-

metic, the second half is all intuitive geometry. In the same series the first half of the ninth is all algebra and the last half is a unit of rigorous demonstrative geometry. Certainly there is variety. In fact it would seem that the possible arrangements have been almost exhausted. Nevertheless, one cannot make such a study without securing much light on the elements to be included. For example, there is unanimous agreement concerning the inclusion or exclusion of certain formulas, principles, topics, and the like, as will be shown in a later table.

Scarcely any critic would acknowledge that these alternative schemes of arranging the materials chosen from arithmetic, algebra, geometry, and the like, are equally effective in forming desirable habits, giving useful information and increasing mathematical power. The placing of a topic in junior high school mathematics that will be reasonably satisfactory involves the teaching of experimental materials and a very careful testing of methods and results so as to disclose the most effective use and organization of materials. For example, the eighth grade book of a well-known junior high school series includes pages and pages of manipulation of symbolism that was condemned in vigorous terms by the National Committee on Mathematical Requirements for use even in the standard algebra courses for the ninth year in a senior high school. The survey of the junior high school books show definitely that it is futile to expect satisfactory text materials to be constructed at a desk remote from pupil activity.

There is here a further effort to secure a list of objectives by examining the newest in current practice. When all is said and done, these junior high school books were written by the various groups under the drive of a new philosophy of secondary education. For some of these groups this may have been quite unconscious, but it probably was operating. There are seven tables that present a bird's-eye view concerning agreements and disagreements on the inclusion of elements and the grade placement of materials. As has been pointed out the content of existing series of texts is not the *sole* criterion of subject matter but it is a "side light," and ONE of the criteria used in the final check of Chapter V.

C. *To Make an Analysis of Materials Used in Schools Undertaking Innovations (so-called experiments) to Discover Significant "Leads" from the Work of Unusually Well-*

Prepared Teachers Who Are Given Almost Complete Freedom from Traditional Demands.

The purpose of this study* was to make generally available the experience of teachers conducting experimental courses in mathematics. The value of this part of the investigation is enhanced by the fact that in general the teachers in these schools have wide experience, sound scholarship in mathematics, and unusual professional training. This part of our program was undertaken at the request of the National Committee on Mathematical Requirements and is included in the final report of the committee. The material includes a detailed description of one or more courses in each of fifteen schools. Not all schools designated as experimental schools felt that they had made a contribution to the teaching of mathematics. It is reasonably certain that no work considered unique by the persons doing it was omitted.

The search for specific outcomes in this field, as in the preceding, does not result in a sole criterion, but there probably is no one who was closely associated with the National Committee on Mathematical Requirements who doubts that this study was a powerful factor in shaping the curriculum proposed by the National Committee. The study furnishes us with significant "leads" helpful in formulating the list of objectives, and, in particular, this study furnished the basis for some of the guiding principles listed in Chapter I.

D. *To Survey the Studies That Throw Some Light on the Selection of Elements Constituting the Subject Matter of These Grades and to Supplement These Studies in Important Sectors of the Field.*

The objective studies dealing with the curriculum constitute the most helpful source now available for the selection of elements. What fractions shall we teach? We may turn to the studies of Woody, Mitchell, Wilson, and others for help. What algebraic processes should be included? If we accept the principle of social usage in its broader sense we can turn to an important study by the National Committee on Mathematical Requirements, and to another by Thorndike. All together there are about 30 helpful studies of this type. These are reviewed or published for the first time in Chapter III.

The studies heretofore available were supplemented (1) by a study purposing to discover the frequency of "uncommon"

*Published in full as Chapter XII in the Reorganization of Mathematics In Secondary Schools.

fractions by means of a newspaper count; (2) by a count of the frequency of geometrical terms, principles, and relations used in materials that pupils of the junior high school age are supposed to read widely.

This chapter attempts to evaluate the most important of the objective studies that deal with the social uses of arithmetic. It also includes a summary table of these studies which constitutes one of the five citeria used in Chapter V. The conclusion of this chapter is that no single available study nor indeed all of them taken together, furnish us a sole and reliable criterion for the selection of a specific list of objectives. Taken as a group, however, they serve as helpful sidelights on the problem.

This summary table is a course of study which we would teach if we decided to teach only those elements for which some kind of a positive case can be made by evidence drawn from objective studies dealing with social use.

E. *To List a Specific Set of Pupil Outcomes for the Junior High School Grades Considered as a Unit.*

The conventional classification of mathematical aims has been (1) disciplinary, (2) cultural, (3) utilitarian. Very few courses of study attempt to show (and probably they couldn't demonstrate) how these aims may be achieved. In fact this classification is too general to be of much help to the classroom practitioner.

Turning to "Cardinal Pinciples of Secondary Education" (page 9) we read "Education in a democracy should develop in each individual the knowledge, interest, ideals, habits and powers whereby he will find his place and use that place to shape both himself and society toward ever nobler ends." This also is too general and needs to be translated into school procedures. We need to ask what definite element of knowledge do we wish pupils to master, what interests to manifest, what specific habits to fix.

The practice of those who construct teaching materials has been to develop a series of topics that hang together and are of some interest to the authors. These are directed toward general aims and seldom, if ever, are specific outcomes considered. There was no evidence, at the time that the program reported in this study was undertaken, of the existence of a single secondary school mathematics book which had been constructed by (1) formulating a list of specific outcomes;

(2) developing topics or units of subject matter and listing pupils activities around these outcomes. The so-called "general curriculum builders" are very, very far from achieving a course of study in mathematics that is built around a list of specific outcomes previously determined. Mr. Bobbitt's Los Angeles study will serve as an illustration.

Our program attempted to put the "horse before the cart." We have tried to secure a list of specific skills, elements of information, concepts and attitudes that determines the selection of materials. This list was subjected to a three-year period of revision simultaneous with the readjustment of materials. The amount of revision of teaching materials due to the control exercised by the list of outcomes may be described quantitatively by the fact that, when the last revision of the experimental materials was made, not a single page of type of the seventh grade book was left standing in the making of the revised book. The evidence is clear that a list of outcomes cannot be formulated by a college professor removed from pupil activities. The outcomes must be tested and modified in actual teaching situations.

Originally, the list was intended to be all inclusive. The sources used were: (1) the indices of five standard arithmetics; (2) the indices of five standard algebras; (3) the indices of five sets of junior high school mathematics books; (4) the objective studies in education dealing with the selection of mathematical materials; (5) the judgments of seventy students in a special methods course at Teachers College (Columbia University); (6) a few items added by the two juries of general educators* and teachers of mathematics**.

The complete list of objectives was checked against the following criteria: (1) the list of material suggested on pages 21 to 27 and in Chapter II of the Report of the National Committee on Mathematical Requirements; (2) the contents of seven series of junior high school mathematical texts; (3) the combination of Guiler's inventory of courses of study in arithmetic and the results of the author's inventory of courses of study in ninth grade algebra; (4) the objective studies in education having a bearing on the selection of curricular materials in mathematics; (5) the judgments of an educational jury and the judgments of a jury of competent mathematics teachers who had not participated in determining the other criteria used.

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*For a list of educators see page 00

**For a list of mathematics teachers see page 00

An element that met three of the five conditions set forth in the five criteria was accepted in the *basic* list of objectives. The objectives rejected for the basic list were included in a *subsidiary* list. Both lists are published in Chapter V.

F. *To Search the Literature of the Philosophy of Secondary Education, of Educational Psychology, and of Committee Reports Dealing with the Mathematics of the Secondary School as a Basis for the Formulation of a Set of Guiding Principles Which Would Assist in the Selection of Subject Matter and Which Would Control the Development of Text Materials.*

As has already been pointed out the construction of our textbook materials has probably suffered most by the practice of an author who sits down at a desk and develops what he thinks are good materials for children without carefully considering criteria by which he can from time to time check his own work. Too often the author has one or more hobbies. This may take the form of a topic in which he is especially interested, or a type of mathematical work that he likes to do. Undoubtedly Wentworth's mind was the kind that liked to factor. The result was pages and pages of factoring. Other writers imitated and so we had a standard of drill work largely on non-essentials set up and fixed on the country for sixty years at least. An eighth grade book with its complex factoring cases and difficult fractions has already been cited. Almost every book has its debauching excesses.

The only available safeguard is a set of guiding principles —hypotheses—against which the author can check day by day. Herein lies one assurance against serious errors, fads, or panaceas. Where shall these principles be found if not in the contributions of modern educational psychology of secondary education as expressed, for example, in our books dealing with the junior high school, Cardinal Principles, and in the articles written by those who have been given special thought to the teaching of mathematics in these early grades? Very much as the surveyor sets his stakes to determine the limits of a new road bed, so the general character of the experimental teaching materials was determined by the formulation of a set of guiding principles that were kept in mind from the writing of the first page to the last revision.

Members of the National Committee, whose final report included some of these guiding principles, lead the discussions in scores of mathematical groups throughout the country over

a period of more than two years. This discussion refined these principles and made them serve a better purpose than they otherwise would. It was exceedingly fortunate for our experimental program that the activities of the National Committee were carried on simultaneously with it.

The bearing of this chapter upon the study of specific objectives must be obvious. After the materials were tentatively selected, there was yet the necessity of a searching check against the principles drawn from the fields enumerated to determine the order of arrangement, emphasis, degree of mastery desired, and method of presentation. Indeed, our technique of making objective investigations is as yet so defective that the principles stated operate in determining the final selection or rejection of materials.

G. *To cooperate in a "Writing-Testing-Revision" Program with the Hope of Refining the List of Objectives for these Grades and to Measure the Effectiveness of the Program by a Series of Tests which Secure Results on One of the Most Important Issues,—the Improvement of Computation with Whole Numbers, Common Fractions, Decimals, Per Cent and Common Units of Measure.*

It may seem that the study should close with Chapter V, but it is fundamental that curricular materials be refined in actual teaching situations under as precise test conditions as modern methods permit. Any program for improvement of teaching materials that falls short of an extensive classroom trial and revision under conditions controlled, as far as they can be, would seem futile. The chapter presents evidence of the value of having a specific set of objectives by showing marked improvement by a number of controlled and tested classes. As has been suggested, the chapter furnishes illustrations of the method of refining the lists of objectives through classroom trial. In addition there is presented a technique of grade placement, and of organizing materials so as to insure more effective learning.

CHAPTER I.

GUIDING PRINCIPLES

Need for List of Guiding Principles—In the moral realm we have the Decalogue, a group of principles of conduct simple and direct. The engineer even in his preliminary survey drives stakes that indicate roughly at least where the road-bed is to be. In education, also, it is important to have guiding principles. It would seem necessary for such a study as is here being undertaken, to list a very brief list of principles to furnish the perspective. What are the four or five or six fundamental principles that will be helpful to the teacher in her daily work when she considers such questions as "What materials shall I teach and how may these be taught best?"

The following pages include a "hexalogue" drawn from the field of Psychology and Philosophy of Education; (1) for secondary education; (2) for the junior high school; and (3) for the mathematics of grades seven, eight and nine. It is important to note that these principles are to be practical working guides for the every-day classroom teacher of the special subject in the junior high school. We are here seeking for a few fundamental principles that will help the teacher to interpret the mass of readings appearing in educational and non-educational journals and prevent her being swamped and disorganized by this volume of material. Obviously such a list of principles is empirical, but the critic has a right to know the foundations of the objective considerations included in the succeeding chapters.

The following is an attempt to state a brief list for each field. The comment with each principle intends to express the important corollaries and implications.

FOR SECONDARY EDUCATION

I. *The Principle of Human Satisfactions: "I am come that they might have life and that they might have it more abundantly." Jesus; John* 10:10.

Thus the greatest of teachers defined the teacher and his task. Independent of its religious association, could there be a more adequate statement of a teacher's attitude towards the

group? Most of the attempts to state the purpose of education do little more than "ring the changes" on this fundamental statement of the abundant life. Certainly the trend at present is to state secondary education in terms of human satisfactions.

The listing of seven objectives in Cardinal Principles of Secondary Education seems to be an attempt to apply this fundamental principle to the present-day school. According to this principle, the first obligation appears to be upon society to enable each individual by developing knowledge, interests, attitudes and habit to realize the maximum human satisfactions.

II. The Principle of Social Progress: Progress of society is not inevitable, but is conditioned by the extent to which the individual recognizes his obligation to shape society toward more worthy aims.

This principle has been stated perhaps most clearly by Dewey* and by the Commission on Reorganization of Secondary Education.** Dewey says that progress "is not a wholesale matter, but a retail job to be contracted for and executed in sections." A clear implication is that after society has done all that it can see to do to develop the personality of an individual, it is then the obligation of the individual to "contract for and execute his sections."

III. The Principle of Habit Formation: Education is a process that society uses to make desirable changes in the individual, largely through habit formation by putting together the things that belong together and achieved through repetition and a satisfying emotional setting.

The beginning of the principle suggests Thorndike|| although both James§ and Dewey† placed great emphasis on the need for making automatic and habitual, as early as possible, as many useful actions as we can. Thorndike has suggested certain machinery in his three laws of Readiness, Exercise and Effect. If space would permit, it probably would be possible to show that all of these laws are included in the principle.

A further implication is the necessity of breaking up subject matter into small enough bits so that the pupil can at all times inform himself as to how well he is succeeding.

Dewey‡ says: "The emotional accompaniment of the progressive growth of a course of action, a continual movement of ex-

*In International Journal of Education. 26:315 f.
**See page 9, Cardinal Principles of Secondary Education.
||See Thorndike, "Educational Psychology," Vol. 1.
§James' "Talks to Teachers," page 66 f.
‡Dewey, "Human Nature and Conduct," page 24.
†See Dewey, "Interest and Effort," page 35.

pansion and achievement is happiness." In still another place he associates the happiness of childhood with habit-forming as an expansion of power.

IV. The Principle of Method: The school must give conscious practice in problem solving.

In this single sentence is expressed Dewey's concept of growth through activity, Kilpatrick's emphasis upon purpose, and the importance of a wider use of sense experience advocated by Eliot.

As early as 1903 Dewey* wrote: "Scientific methods, the methods pursued by the scientific inquirer, give us an exact and concrete exhibition of the path which intelligence takes when working most efficiently, under most favorable conditions. What is primarily required for that direct inquiry which constitutes the essence of science is first-hand experience; an active and vital participation through the medium of all the bodily organs with the means and materials of building up first-hand experience. Secondly, in that freed activity of mind which we term 'science' there is always a certain problem which focuses effort, which controls the collecting of facts that bear upon the question, the use of observation to get further data, the employing of memory to supply relevant facts, the calling into play imagination to yield fertile suggestion and construct possible solutions of the difficulty."

Problem solving in this sense is genuine research for the pupil at his level. More extensive units of thought (McMurry) will be undertaken. There is purposing, planning, consulting, co-operation, sharing and checking by children. Correlation, motivation and the socialized recitation all fit into the scheme of things. Some subsidiary implications are: using the school day as a working period, developing a type of school room furniture different from the conventional, securing a teacher well enough trained to keep out of the way, and managing so as to get longer recitations and a longer school day.

Finally, if the principle is true, subject matter is not so important—not what the pupil learns but what happens to him while learning is the vital factor.

V. The Principle of Individual Differences: There are enormous differences in capacity and interests between members of an ordinary class.

As a by-product of the educational measurement of Thorndike, Ayers, Courtis and others, the facts of individual differ-

*Elementary School Teacher, 4:193-204.

ences have been clearly demonstrated. It is not unusual to find a class even when every effort has been made to obtain homogeneous grouping in a special subject, in which the upper tenth of the class can do from three to six times as much as the lowest tenth either on new materials or simple skill problems.

The principle implies that there is no such thing as a *class* of twenty pupils. It is a group of twenty unique individuals—each demands a special type of curriculum and a diagnosis of individual disabilities.

It is not clear that the principle demands the Dalton or a similar plan, but the trend toward individualized instruction is unmistakable. It cannot be said too often that Miss Brown is teaching not a class, not algebra, but Bobby Jones and Mary Smith.

VI. The Principle of Measurement: A modification of the stimulus changes the response and that difference can be measured.

In this statement we have the suggestion of Thorndike's "S-R Bonds" and the faith that the product of education can be measured. The teacher who says to herself before school: "It makes a difference what I do today and I shall try to find out how big this difference is" probably is a live and growing teacher. She will be stimulated to become a student of her learning problems.

FOR THE JUNIOR HIGH SCHOOL

I. The Principle of Minimum Essentials: Pupils should learn "to do better those desirable things that they will perform any way."

This principle demands careful study of the pupil as an individual. It means taking an inventory as well as we can, of his capacity and of his interests. It implies an emphasis upon minimum essentials and a careful study of a community's needs. As a concrete illustration, we may note that a pupil will read newspapers and magazines. It follows that the junior high school should see to it that he controls the fundamental skills of reading.

*II. The Prinicple of Standards: The junior high school should "reveal to pupils higher types of activities and make them both desirable and to an extent, possible."**

The junior high school will fail if it does not insist on a fine quality of workmanship. It must furnish a good model on

*Briggs, T. H., "The Junior High School," page 157.

every occasion. Perhaps equally important is the school's obligation to supplement the life activities of an individual by better forms. For example, the pupil is surely going to read and he is going to hear music. What is he going to read and what music will he enjoy? The school, by giving him desirable types of experience, may lead the pupil to appreciate and to demand higher standards.

III. The Principle of Exploration: The junior high school should offer a basic, introductory course in each of the chief fields of human activity, Art, Language, Mathematics, Science, and the like, for the purpose of self-discovery and guidance.

The beginning of the junior high school takes the pupils when most of the fundamental skills have been given some attention. A decreasing number of pupils drop out of the junior high school. If there are common ideals to be established, the junior high school period would seem to be the desirable time. The notion of the junior high school as a people's school, a sort of introduction into the liberal arts is certainly spreading. There is excellent evidence that even for those who continue, a general basic course is to be desired. Certainly it is important that those who drop out early shall come in contact with the simple and significant material in each field.

It may also be urged that a survey course offers opportunity for exploration. For example: the pupil in a general mathematics course does not face the choice, algebra or no algebra, when in fact he hasn't the slightest idea of what algebra is about. In a general mathematics course he is given enough of a touch of algebra in grades seven and eight so that in later years he has some notion of the type of learning involved. In brief, the exploration can well come *within* the basic courses.

Finally, careful observation of the individual pupil in basic courses should furnish valuable material for guidance. It appears that even seniors in college life need advice on life choices and certainly the secondary school should not neglect this important function.

IV. The Principle of Present Values: Each course in the junior high school should be planned with the purpose of giving each pupil the most valuable training he is capable of receiving that year with little reference to later courses.

In particular senior high school courses and college entrance requirements should not determine the selection of materials. There probably is no conflict between the needs for common life and college demands. If anybody needs a broad basis with

an emphasis on the fundamentals, it is the future college man. But the requirements for college, formally stated for pupils from three to six years more advanced than junior high school pupils cannot be accepted as valid objectives for the junior high school. As Briggs states it, each course "should be so arranged as to be most worth while at the time taken and to the extent it is pursued."

V. *The Principle of Specific Objectives: Each one of the large fields of human activity in the junior high school should list the specific habits, attitudes, concepts and items of information which are to be used as objectives by the teacher.*

In swinging away from the formal traditional courses to the general exploratory courses there is a real danger of drifting to low mastery. There is a great variety in the courses of study for a specific grade in any special field. When teachers get each other's products or when the pupils go to college, the situation usually involves considerable confusion. Describing the typical secondary school product, the philosophic comedian Will Rogers says "literally they don't know nothing." This seems to be the attitude also of many professors of freshman courses in college. We can be and must be quite as definite in our aims as were those who formulated the traditional courses. Certainly there was not much doubt on the part of the teacher of algebra between 1890 and 1900 as to what he was expected to teach.

It should be possible to list our objectives precisely and still leave the teacher wide opportunity for planning how these objectives may be achieved.

VI. *The Principle of Practice in Citizenship: The junior high school must offer opportunity to children for learning to live together.*

In this complex age social adjustment is one of the difficult problems of an individual. Dewey pointed out the futility of schools preparing for life with an insistence that education is life. Hence, we have had great emphasis in the literature of secondary education on "school as life here and now." We need to go a bit further and assert "not for life here and now," but for life *as we would have it.*

Surely the junior high school dare not train its community citizens for efficiency in political activity as it is. The school must be an exponent of higher standards of citizenship than those now in practice. Even if later boys and girls should be disillusioned and be required to adjust their philosophy, they probably would look back with longing to this period of idealism and refer to this time as the "good old days."

FOR THE MATHEMATICS OF THE JUNIOR HIGH SCHOOL

I. The Principle of Immediate Values: The course in each year should give the pupil the most intrinsically valuable mathematical information and training which he is capable of receiving at that time, with little consideration for the needs of subsequent courses.

This principle is obviously only a corollary of the fourth principle in the preceding list. In substance this statement was accepted as one of the fundamental principles in the Report of the National Committee.* In practice the principle means that the first year of the junior high school shall give emphasis to those principles that are most valuable for the pupil at that level with little consideration for later courses.

II. The Principle of Psychological Organization: The content of these courses must be selected from the point of view of the children and from the consideration of social needs and not solely from the logical requirements of mathematics.

Mathematics as traditionally organized is a logical structure. It represents mature thinking, or that of one "who has been over the road." It is here proposed to emphasize the interrelations of the various topics in the different fields, which involves cutting across the logical units.

A practical suggestion is that the material shall be vitalized through the early introduction of elements that are commonly delayed until later years. It is proposed, also, that mathematics shall make some contribution to the other school subjects, as for example, elementary science, industrial arts and fine arts, and that these subjects shall in turn lend meaning to mathematics. A third suggestion is to make the course more graphic. The unity of space and number should persist throughout for geometry is a graphic method furnishing a concrete source, setting and illustration of significant number relations. Hence, measurement is one of the fundamental processes in which the pupil may discover important number relationships directly through the senses.

*See page 14, The Reorganization of Mathematics in Secondary Education. It appears also exactly as given here on page 237 of the same report.

In brief, if a graphic sketch can be made of a situation dealing with number, more people will be likely to understand the principle.

III. The Principle of Content: To achieve the aims implied in the preceding principles the junior high school course should include the simple and significant elements of arithmetic, intuitive geometry, algebra, statistics and numerical trignometry.

The mathematics of the junior high school will probably mark the end of the required mathematics and hence the junior high school course must throw into bold relief those general mathematical ideals, tools and habits which are now regarded as of maximum importance. Manipulation as an end is to be eliminated. Mechanical work can be justified only when necessary for understanding and using fundamental principles.

IV. The Unifying Principle: Throughout the course the idea of relationship, or of dependence of one quantity upon the other is to be emphasized.

As far as is known the newer courses have not been criticized by competent writers as lacking unity. It is clear that unity and rigor of learning do not demand a logical organization from beginning to end. From the mathematical point of view the notion of function is the unifying principle. Even this has to be safeguarded a bit for from the point of view of teaching; the basic guiding principle is not found within the science itself, but within the children's purpose.

In organizing the course the usual emphasis upon the special divisions of mathematics and the customary time allotments should be replaced by the introduction of the topics in such a way as to insure a maximum of direct and intensive application, flexibility and significant inter-relations. The material is to be socialized through the extension of units of instruction from classroom exercises and topics through a series of activities, projects or problems requiring: (1) co-operation; (2) the sharing of interests, efforts and results.

V. The Principle of Mastery: The junior high school course must result in a marked increase in the accuracy of computation with integers, decimal fractions, per cents and certain common fractions.

It is not likely that the public will indefinitely be satisfied with the low degree of mastery exhibited in the test results to be presented in the next chapter. In all probability we shall

achieve better results through individualized instruction of elements that we wish to raise to a skill level.

VI. The Principle of Social Utility: Junior high school mathematics should teach the necessary social and economic uses of arithmetic.

In the light of the objective evidence it is almost certain that these uses are very few in number. Moreover, we need to recognize that, for many of the forms now taught, the children have neither experience nor maturity. It is a fallacy to assume that because an adult has to do a problem in stocks and bonds that this is real, vital material for a twelve or thirteen-year-old girl. We shall probably be sensible if we try in such cases to get nothing more than a little understanding.

It is not apparent that recent comers among our immigrants have much more trouble with the simple business forms than people who go through our schools. It is possible that a little drill in a natural setting when the problem is met later in life goes a long way in achieving the necessary skill. Such forms of business practice as insurance, brokerage, stocks, bonds, and the like, should be placed late in the course to use the greater maturity, experience and mathematical knowledge of the pupil, always remembering that whatever we wish to teach must be taught while the children are still in school.

CHAPTER II

AN INVENTORY OF THE MATHEMATICAL EQUIPMENT OF BEGINNING SEVENTH GRADE PUPILS AND THE DEGREE OF MASTERY OF CERTAIN TASKS IN ALGEBRA

What does the first six years of arithmetic instruction achieve? It seems difficult to believe that we have constructed curriculum materials for advanced grades in a subject like mathematics without inquiring as to the strength of the foundation upon which we propose to build. Probably we never can make a complete inventory, but we can get facts sufficiently important to greatly modify our practice. In order to get some light on the proposed question, an inventory test was used and a brief compilation made of results secured through the use of well known existing tests.

The test, which appears on the following pages, was constructed by John R. Clark, Harold Rugg, and the writer. The criteria of the test elements are: (1) appearance in courses of study; (2) the judgments of writers of arithmetic.

For the first condition Guiler's course of study* is accepted as reflecting practice in the first six grades. Any task appearing in Guiler's list for any grade is almost certain to have received intensive teaching in many schools. The tasks that appear somewhere in Guiler's course of study for the first six years are not marked with an obelisk in the inventory test.

It was thought desirable also to make use of the judgments of persons presumably competent to state outcomes of instruction, namely, the writers of arithmetics. Consequently, the three designers of the test submitted a tentative form of the test to twenty-nine authors of arithmetics with the double request: "(a) List here arithmetical elements which you think beginning seventh grade children may know, and which are not included in our list. (b) List here elements included in our list, but which you are convinced beginning seventh grade children in general do not know." Most of these writers were friends of one or more of the three men submitting the request;

*Included in a doctor's dissertation filed at the University of Chicago.

hence, the response was most gratifying. All but one* responded to the test. The returns varied all the way from a brief line of hearty commendation by Professor Thorndike to several pages of carefully considered notes by Professor George Myers. On the whole, these returns were helpful though they did not in fact modify the test to any considerable extent. The test was then revised on the basis of the judgments secured.

The test was divided for administrative reasons into two sections. The first section numbered 1 to 69 inclusive. The second section numbered from 70 to 125 inclusive. The teachers could take the whole recitation to each section and thus make certain that there would be no complaint of insufficiency of time. The directions given to the pupil were: "When you turn this page, you will see some exercises in arithmetic. Read each of these problems, think about it carefully and write the answer in the blank space. The answers to some exercises are numbers, while to others the answers are words or drawings. Take the exercises in order. If you find an exercise that you cannot do after thinking carefully, go on to the next. Since you will have plenty of time, there is no need to hurry." The papers were collected at the end of thirty minutes, but it will be noted that pupils were not urged to speed by being warned beforehand of a time limit. It is believed that the test represents practically an untimed test. The writer himself administered this test to three hundred twelve children in New York City. In every test nearly all pupils finished. In one case six out of one hundred twenty-three pupils not finished within the thirty-minute period were given an extra half hour. The gains in the number of correct responses by these six children were 0, 0, 1, 2, 4, 5. Only two pupils improved their scores by answering the same problem correctly in the over-time. The possible spread of gain in the over-time and the low number of pupils who do not finish within the time limit suggests that the mass results reported in this study would probably be affected very little by a marked extension of time. Indeed, is it probable that pupils selected at random from those completing the test might improve their scores as much by being forced to give more careful attention to the test in an over-time period as did the six pupils cited above.

The following table reports the results secured from 3260 beginning seventh grade pupils of our co-operating** schools.

*The late George Wentworth, Jr., who died before he could carry out his promise to contribute to the test.

**See Chapter VI for an explanation of the term "Co-operating."

The table includes the elements in the inventory test and also the percentage of correct responses for 3260 pupils on each element.

The table should be interpreted as follows: Of the 3260 pupils tested, approximately 98.1% or 981 out of every 1,000 children tested responded correctly to the first task—"One dozen equals how many things?" Of the 3260 pupils tested, approximately 68.4% wrote the correct response to exercise 77, namely, "Write one-half as per cent." The results are given accurately to the nearest tenth of one per cent. Thus, in the case of the first problem, 3198 pupils wrote the correct result.

	Test Elements*	Percentage of Correct Responses
1.	One dozen equals how many things?	1. 99.0
2.	One pound equals how many ounces?	2. 96.2
3.	One yard equals how many inches?	3. 87.5
4.	One ton equals how many pounds?	4. 77.9
5.	One gallon equals how many pints?	5. 75.6
6.	One bushel equals how many pecks?	6. 81.7
7.	One square foot equals how many square inches?	7. 60.3
8.	One mile equals how many feet?	8. 58.2
9.	One cubic foot equals how many cubic inches?	9. 48.1
10.	One acre equals how many square rods?	10. 45.1
11.	One hour equals how many minutes?	11. 96.6
12.	One pint equals how many cupfuls?	12. 56.9
13.	One cubic yard equals how many cubic feet?	13. 44.7
14.	One bushel equals how many cubic inches?	14. 10.6
15.	One bushel of wheat equals how many pounds?	15. 22.0
16.	One gross equals how many things?	16. 58.1
17.	One minute equals how many seconds?	17. 95.1
18.	April has.....days, July has.....days; November has.....days; August has.....days.	18. 55.0
	PART II	
19.	If you have two numbers, how can you find their sum?	19. 93.6
20.	If you have two numbers, how can you find their quotient?	20. 88.3
21.	If you have two numbers, how can you find their product?	21. 85.0
22.	If you have two numbers, how can you find their difference?	22. 88.0
23.	If you have the difference between two numbers and the smaller one, how can you find the other number? Write here	23. 43.9
24.	If you know the length of the sides of a rectangle, how can you find the area? Draw a line under the answer. By adding; by subtracting; by multiplying; by dividing?	24. 71.6

*This list of 125 exercises are the elements of the Schorling-Clark-Rugg Inventory Test. The form printed for children's use is necessarily different from that given here. The test was constructed for investigational purposes only.

	Test Elements		Percentage of Correct Responses
25.	Can we always multiply both numerator and denominator of a fraction by the same number without changing the value of the fraction? Write Yes or No here	25.	23.4
26.	Can we always subtract the same number from both numerator and denominator of a fraction without changing the value of the fraction? Write Yes or No here	26.	22.6
27.	Can we always divide both numerator and denominator of a fraction by the same number without changing the value of the fraction? Write Yes or No here	27.	5.2
28.	Can we always add the same number to both numerator and denominator of a fraction without changing the value of the fraction? Write Yes or No here	28.	0.4
29.	Moving the decimal point one place to the left in a number; ....the number by 10	29.	42.1
30.	If you have the quotient of two numbers, and the smaller number, how can you find the other number? Write here	30.	47.9
31.	If you have the sum of two numbers and one of them, how can you find the other number? Write here	31.	67.9
32.	If you have the product of two numbers and one of the numbers, how can you find the other number? Write here	32.	44.5
33.	If you know the lengths of the sides of a drawing, how can you find its perimeter? Write here	33.	48.4
34.	Moving the decimal point one place to the right of a number; ....the number by 10	34.	49.5
35.	If you know the cost of a certain number of pencils, how would you find the cost of one of them? Write here	35.	82.9
36.	What is the average of 4, 6, 8 and 10? Write here	36.	42.7
37.	Mr. Edson bought three Fords for $1,500 and sold them for $1,200. What was the cost per Ford? Answer	37.	58.7
38.	Mr. Edson bought three Fords for $1,500 and sold them for $1,200. What was the selling price per Ford? Answer	38.	55.0
39.	Mr. Edson bought three Fords for $1,500 and sold them for $1,200. Underscore the correct parts in the following: Mr. Edson (gained, lost, $1,200, $1,500, $100, $500, $300, $2,700) on each Ford	39.	31.9
40.	Does 4896÷10 equal 4.896, or 49.96 or 4896 or 489.6? Draw a circle around the right number	40.	35.5
41.	Does 1.2 x .5 equal 6.0, or .60, or .060, or 60? Draw a circle around the right number	41.	63.7
42.	Write in figures. Fifty-nine and three hundredths. Write here	42.	76.1
43.	Write in figures: Two million seven thousand six and eight hundredths. Write here	43.	22.3
†44.	Write in figures: MDCCCXXIV. Answer	44.	20.8

PART III

Write the answer to each of the following in the blank spaces at the right of the question:

45.	What is 12 divided by 3? Ans	45.	91.1
46.	What is the difference between 8 and 5? Ans	46.	87.1

	Test Elements		Percentage of Correct Responses
47.	Look at this: $16 \div 8 = 2$. Which number is called the divisor? Ans.	47.	76.2
48.	Look at this: $16 \div 8 = 2$. Which number is called the dividend? Ans.	48.	71.3
49.	Look at this: $16 \div 8 = 2$. Which number is called the quotient? Ans.	49.	82.3
50.	Look at this: $9 + 7 = 16$. Which number is called the sum? Ans.	50.	85.3
51.	Look at this: $5 \times 4 = 20$. Which number is called the product? Ans.	51.	78.4
	†Any task not definitely found in Guiler's course of study (see doctor's dissertation filed at the University of Chicago) will be marked with an obelisk.		
52.	Look at this fraction: $\frac{4}{5}$. Draw a circle around the part of the fraction which is called the denominator.	52.	41.2
53.	Look at this fraction: $\frac{4}{5}$. The number 5 is called the .....of the fraction	53.	62.9
54.	Fill in the blank space in the following sentence: The circumference of a circle means the distance..... .....the circle	54.	68.9
†55.	What is the least common multiple of 5 and 3? Ans.	55.	48.5
56.	Fifty per cent of a number is the same as what part of that number? Ans.	56.	71.4
57.	Twenty-five per cent of a number is the same as what part of that number? Ans.	57.	67.2
58.	% means	58.	51.4
59.	Look at this: $7 - 3 = 4$. Which number is called the minuend? Ans.	59.	35.9
60.	What is the square root of 16? Ans.	60.	30.0
61.	Look at this: $7 - 3 = 4$. Which number is called the subtrahend? Ans.	61.	42.8
62.	Babe Ruth buys 5 bats at $8 each. He finds the total cost by multiplying. Which number is the multiplicand? Ans.	62.	9.4
63.	In Ex. 62, which number is the multiplier? Ans.	63.	14
64.	Write three factors of 24. Ans.	64.	57.0
65.	What must you do to find $\frac{2}{3}$ of $\frac{5}{8}$? Do you add? If not, what must you do? Ans.	65.	54.9
†66.	Make an equation out of the numbers 2, 5 and 7. Ans.	66.	11.9
†67.	What is the second power of 5? Ans.	67.	9.5
†68.	What does 6^2 mean? Ans.	68.	12.3
†69.	What does bh mean? Ans.	69.	6.9
	PART IV		
70.	Write 3% as a decimal. Ans.	70.	85.3
71.	Write $6\frac{1}{2}$% as a decimal. Ans.	71.	64.4
72.	Write .4 as per cent. Ans.	72.	35.8
73.	Write .025 as per cent. Ans.	73.	37.2
74.	Write $\frac{1}{5}$ as a decimal. Ans.	74.	66.6
75.	Write $\frac{1}{8}$ as a decimal. Ans.	75.	71.4
76.	Write $\frac{3}{4}$ as a decimal. Ans.	76.	74.8
77.	Write $\frac{1}{2}$ as percent. Ans.	77.	73.5
78.	Write $\frac{1}{4}$ as percent. Ans.	78.	74.1
79.	Write $\frac{1}{8}$ as percent. Ans.	79.	70.1
80.	Write $\frac{1}{3}$ as a decimal. Ans.	80.	62.9
81.	Write $\frac{1}{16}$ as a decimal. Ans.	81.	55.4

	Test Elements		Percentage of Correct Responses
82.	Write $\frac{1}{12}$ as a decimal. Ans.	82.	35.6
83.	Write .25 as a common fraction. Ans.	83.	81.3
84.	Write .125 as a common fraction. Ans.	84.	47.5
85.	Write .06¼ as a common fraction. Ans.	85.	27.2
86.	What is 3% of 200? Ans.	86.	59.7
†87.	The ratio of 1 to 2 is equal to the ratio of 5 to	87.	33.0
88.	The interest on $200 for 3 years at 4% is $	88.	60.8
89.	Fred Stone found that 5 percent of 200 is 10. What number is the percentage? Ans.	89.	19.3
90.	Fred Stone found that 5 percent of 200 is 10. What number is the base? Ans.	90.	48
91.	Theodore Roosevelt was born October 27, 1858, and died January 6, 1919. His age was .....years, ...months, ...days.	91.	29.7

Part V

Make a drawing to the right of each of these words or phrases to show that you know its meaning.

92.	Circle	92.	97
93.	Parallel Lines	93.	78.1
94.	Triangle	94.	85.2
95.	Square	95.	90.8
96.	Rectangle	96.	59.8
97.	Angle	97.	51.3
†98.	Cone	98.	59.4
99.	Right Angle	99.	43.0
†100.	One line perpendicular to another line	100.	26.9
101.	Diameter	101.	43.1
†102.	Arc	102.	28.7
†103.	Radius of Circle	103.	25.6
†104.	Diagonal	104.	16.4
†105.	Quadrilateral	105.	20.2
†106.	Hexagon	106.	12.7
†107.	Sector	107.	10.8
108.	Parallelogram	108.	20.1
109.	Altitude of Triangle	109.	20.1
†110.	Trapezoid	110.	12.2
111.	Cube	111.	37.1

Part VI

†*112.	The following diagram shows how the American Relief Fund was spent. How many dollars were spent in Armenia? Ans.	112.	42.4
†*113.	Complete the following: Armenia received a little more than .....times as much as Russia	113.	35.8
†*114.	The following diagram shows the average family's share in a city budget. How much did the average family pay in 1908? Ans.	114.	24.7
†*115.	When did the city budget increase most rapidly? Ans.	115.	36.7
†*116.	A drawing like that which shows the growth of a city budget is called a	116.	18.7

Part VII

*117.	Underscore the right name in the following sentence: Figure 1 shows a (promissory note, check, account, receipt, letter, sales bill)	117.	43.4

Test Elements*		Percentage of Correct Responses
*118.	Underscore the right name in the following: Figure 3 shows a (promissory note, check, account, receipt, letter, sales bill) ..... 118.	36.3
*119.	Underscore the right name in the following: Figure 2 shows a (promissory note, check, account, receipt, letter, sales bill) ..... 119.	30.0
*120.	Who had money deposited in the Farmers' Bank and Trust Co.? Ans..... 120.	60.0
*121.	Who gave the receipt shown on this page? Ans..... 121.	40.6
*122.	Who received the money mentioned in the receipt? Ans..... 122.	41.7
*123.	Who promised to pay money? Ans..... 123.	47.4
*124.	Who had received the money mentioned in the note? Ans..... 124.	21.5
*125.	When did she promise to pay it back? Ans..... 125.	39.1

Before drawing any conclusions let us look at a few typical results. Exercise 40 reads: "Does 4896 ÷ 10 equal 4,896, or 48.96, or 489.6, or 4.896? Draw a circle around the right number." This required the choice of the right result from four suggested responses. The exercise received 34% correct responses, or approximately one-third of the pupils tested chose the correct answer. Exercise 74: "Write 1/5 as a decimal fraction." This exercise received 59.4% correct responses. Approximately three out of every five children were able to do this simple task. More definitely, in a class of thirty pupils in the seventh grade on an average as many as twelve pupils failed on this exercise. Exercise 83 reads: "Write 0.25 as a common fraction." The table shows that only approximately seven out of every group of ten pupils write the correct response. Exercise 86 reads: "What is 3% of 200?" The number of correct responses is 45.5%—considerably less than half. Exercise 91 required the finding of the difference between two dates—Roosevelt's age at death. Less than one-fifth wrote the correct result. Such illustrations as these suggest that *there are many very simple elements in arithmetic to which the schools give much drill that nevertheless are learned by but a small factional part of the pupils.*

THE SUMMARY TABLE

The Table on the next page represents a summary of the results of the inventory test. The table should be interpreted as follows: There were only eight tasks in the inventory test that as many as 90% of the pupils tested could do. There were

*For diagram and illustrations of exercises starred, see copy of the Schorling-Clark-Rugg Inventory Test, Gazette Press, Yonkers, New York.

twelve exercises that received from 80% to 90% correct responses. There were five tasks so difficult that less than 10% of the pupils succeeded in each.

SUMMARY TABLE

Per cent	No. of Elements
90–100	8
80– 90	12
70– 80	14
60– 70	10
50– 60	14
40– 50	22
30– 40	14
20– 30	14
10– 20	12
0– 10	5
	125

To those interested in learning, the data of this table suggest some significant questions:

1. *What is the Character of the Task that Pupils Succeed in Doing With a High Degree of Mastery?*

There are only eight items which receive 90% correct responses or better, and these items are the simple tasks: "One dozen equals how many things? One pound equals how many ounces? One hour equals how many minutes? One minute equals how many seconds? If you have two numbers, how can you find their sum? What is $12 \div 3$?, and make drawings showing that you know the meaning of the words 'circle' and 'square'." At least four of these items are learned by a considerable number of children before they enter the first grade.

It does not seem that 80% mastery is an unreasonably high standard to expect on items that the school has definitely included in its program, but if we are willing to accept 80% as our standard, we get only twelve additional tasks done at this level. These are: "One yard equals how many inches? One bushel equals how many pecks? If you have two numbers, how can you find their product? If you have two numbers, how can you find their difference? If you know the cost of a certain number of pencils, how can you find the cost of one of them? What is the difference between eight and five? Look at this: $16 \div 8 = 2$, which number is called the quotient? Look at this: $9 + 7 = 16$. Which number is called the sum? Write 3% as a decimal. Write 0.25 as a common fraction. Make drawing to show you know the meaning of the word,

'triangle'." The reader will agree that all of these are simple tasks. Many children learn at least three of these before entering the first grade and it is probable that a considerable number learn all but two of these before completing the third grade.

The table shows that there are fourteen tasks that receive accurate responses from 70% to 80%, and ten items of the test from 60% to 70%. A study of the table will show that all the items down to a 60% standard of accuracy are still very simple.

What happens to the strength of these bonds? The pupils have just completed their sixth grade arithmetic. There are eighty-one exercises out of one hundred and twenty-five that get less than 60% correct responses. Unless these tasks get further drill in school or in life situations out of school, many of these bonds will be greatly weakened. At any rate, one may ask whether anything learned to the point where less than six out of ten children respond accurately while the material is fresh in mind is worth very much to society. For a more complete discussion of this issue the reader should refer to Morrison.*

2. *What Results Could Be Secured with No Formal Instruc- in Arithmetic?*

Since all the tasks above the 60% level are relatively simple and of the type which receive considerable drill in the home, at the corner drug store, in the grocery store, on the playground, and in numerous other life situations, what would be the strength of these connections if pupils were not subjected to formal recitations of textbooks in arithmetic? In other words, how much arithmetic would children learn if no formal study of the subject were attempted? The simplicity of the tasks above the 60% mastery suggests that all pupils with I. Q. of 110 or better would certainly master nearly all of these tasks by the beginning of grade seven. The I. Q. of 110 is chosen, as this figure, according to Thorndike, would include about all the pupils who finish high school with a reasonable degree of comfort.

We should expect considerable gain when a school attempts to pattern its activities after normal child life. Concerning this point a little evidence is available. Professor Junius

*See The Mastery Notion, "The Teaching Technique of the Secondary School," Chapter II.

Meriam* organized a curriculum for the elementary school of the University of Missouri about four studies: namely, *observation, stories, playing* and *hand work.* The significant point relating to this study is that Meriam's school did not include formal arithmetic. Indeed, it is believed that "the best way to teach arithmetic is not to teach it." Four questions are suggested.

(a) How much arithmetic do people learn incidentally through their social relations in school, in the home, at recreational affairs? The number of business men (self-made men) with practically no training in arithmetic, who nevertheless are keen and efficient in the arithmetic of life, is significant. This is the issue which Meriam and his followers raise. The problem needs careful investigation and critical analysis by an unprejudiced group.

(b) How much of the material ordinarily emphasized in arithmetic is really needed? Many teachers insist that much time in arithmetic is taken up in wasteful reviews and in the teaching of meaningless processes. This matter, too, cannot be settled except by a carefully measured experiment. Some light may be obtained by giving a comprehensive inventory test to business men, professional men, and to freshman students in college, so as to ascertain what bonds have been weakened by disuse. If a considerable part of arithmetic is useless, then naturally there would be no great difference when pupils go to their algebra without this subject matter.

(c) How much arithmetic (independent of utilitarian values) does a pupil need to know to succeed in algebra? It may turn out that success in algebra depends less upon success in arithmetic than is commonly believed. This also requires investigation.

(d) How much does the teaching of some task within its natural setting decrease the number of repetitions necessary to give a mental connection a desired degree of strength? Those who advocate the project method would probably assert that drill can be considerably decreased when a principle is taught or a skill is fixed under the conditions that are highly motivated. It will be noted that most things in the inventory test that are reported with anything like a satisfactory degree of mastery are certainly the things that get a great deal of drill in normal child life and get this drill in natural settings.

*"Child Life and the Curriculum," Chapters XIII to XVI.

3. *Are We Satisfied with Low Standards?*

Many European visitors to our schools, though they may be enthusiastic about many excellent characteristics, are vigorous in their indictment expressed in such phrases as: "You foster half learning;" "You are satisfied with low standards;" "You do not fix habits;" and, "There is no question that you can ask your children that any considerable number of them will know." It is not clear what is meant by "half learning." Perhaps it means that half of the children learn all the things that we try to teach them, or that half of the things are learned by all the children. The table suggests that neither happens. A few of the items in the test are learned by nearly all the children, a few elements are mastered by only a few children, and a large number of tasks (the mode) falls between a 40% to 50% mastery. In fact, the inventory test resulted in something very close to a normal distribution. By way of summary, it may be stated that the total number of tasks getting 50% correct responses is 58 out of 125—less than half the test. If by half learning is meant that each and every one of half the number of outcomes is mastered by less than half the pupils, there is evidence that the criticism is none too harsh. In fact, we do not achieve anything like half learning in this sense. We are driven to admit that pupils lack mastery of the essentials of arithmetic.

4. *How General is This Characteristic of Low Mastery?*

Many of the teachers in the cooperating schools were students in summer school courses. They represent the type of teachers who make sacrifices to improve their results and who are interested in tests and experiments, in general are a highly selected group, probably far above the average. It is reasonable to believe that results obtained by them will be the work of pupils taught under superior conditions and by methods representing good practice. The test reveals a dark picture, but dark as it is, it probably is much too bright. The test has been extensively used and many of the data collected are not reported in this study. As the test is being extended, particularly to the smaller towns and rural conditions, the percentage of correct responses tends to be lowered throughout the test.

5. *Does the Same Condition of Low Achievement Hold in Ninth Grade Algebra?*

On this point we can speak quite definitely for Thorndike has reported a similar study on the strength of algebraic con-

nections.* He reports on twenty-eight tasks. His table includes records secured from ten schools though the number of pupils tested is not given. The following table shows the results secured on six of the tasks by the four lowest schools. The percentages given are those for correct responses.

Modified from Thorndike's Table	Percentage of Correct Responses			
(4) From $5a-b-2c$ subtract $3c-3a$	49	68	41	35
(7) $7d \times 2de^2$	74	70	72	74
(9) $8-5\ (d+2)$	71	39	20	37
(17) If $a=2$ and $b=3$ what does $5a-2ab$ equal?	68	56	38	49
(26) $15=7W-4$ what does W equal?	67	45	42	51
(27) $\frac{V}{4}=a-2$ what does V equal?	51	38	26	26

It is true that some of the preceding tasks might be criticized as including subleties that are not ordinarily touched by drill. Thus the arrangement in Exercise 4 adds to the difficulty. A direction or a challenge of some sort could well have been added to Exercise 9. Teachers do not give much practice on such equations as are represented in Exercise 26, and the statement for Exercise 27 could probably be improved. Indeed, it is likely that Thorndike used these subleties to demonstrate to teachers the need for more varied drill. On the whole, it must be admitted that these exercises are very simple when contrasted with the complex manipulations required by a standard ninth grade algebra course. Thorndike concludes, "It does not seem an exaggeration to say that on the whole these students of algebra had mastery of nothing whatsoever. There was literally nothing in the test that they could do with anything like 100% efficiency." This is a severe statement of the results secured from high school people who are undertaking their second year of algebra.

The results secured by Hotz's test furnish additional evidence. The following table consists of elements selected from the table which appears in Thorndike.* Here also the table has been modified to include percentage of correct responses. The pupils tested had studied algebra for nine months.

*"The Psychology of Algebra," Chapter XII.

RESULTS ON THE HOTZ TEST

	Percentage of Correct Responses		Percentage of Correct Responses
$\frac{3c}{4}-\frac{3c}{8}=$	64	In the formula RM = EL find the value of M	51
$(-3xy^3)^4$	62		
$\frac{y}{3}=\frac{5}{2}=\frac{y}{4}$	70	The area of a circle = r^2 in which r = radius of the circle and π = 3 1/7. Find the area of a circle whose radius is 7 ft.	48
$\frac{2x}{3}=\frac{5}{8}$	80		

The particular elements selected are very much easier than the exercises ordinarily expected of ninth grade algebra students at the end of the year. Here, as in the Thorndike results, we find a very low degree of mastery. The situation revealed by the inventory test for arithmetic is probably worse in ninth grade algebra, for pupils get little practice on these algebraic skills in out-of-school activities. The results of Thorndike, Hotz, Childs, and Monroe, all support the belief. By way of summary, we may quote Thorndike, "Complicate the situation slightly and the pupils fail."

6. *How Significant is the Small Percentage of Correct Responses on a Series of Arithmetic Tasks?*

It is probable that if one hundred graduate students with "B" standing, or better, were asked one hundred questions such as: (1) Who invented radio? (2) Who was the influential Chinese delegate to the Hughes Disarmament Conference? (3) To whom are we indebted for the American automobile? (4) Who was the vigorous South African delegate to the conferences that organized the League of Nations? (5) What use do insurance men make of the slogan "54-36-5-4-1? that the number of correct responses would be very low. It is also likely that a set of questions concerning familiar facts and principles from courses in history, geography, or literature would yield depressing results. One may fairly ask, "Is not the condition, the low degree of mastery revealed in the preceding tables, generally true in all learning situations?"

But our inventory test results are not comparable to these general illustrations. We are dealing with the essentials of a subject concerning which society for generations has had fairly definite agreements. Week after week and year after

year some of these tasks are given systematic drills to fix these skills. Society spends from one-fourth to one-fifth of its school funds for the intermediate grades in the teaching of arithmetic. This is a very great investment on the part of society and there should be more definite returns.

7. *How Well Can Children do Arithmetic Tasks that are not Included in Textbooks or Courses of Study?*

It will be recalled that some of the tasks were added to the preliminary edition of the inventory test upon the advice of writers of arithmetic. They believe that certain tasks should be known at the beginning of the seventh grade even though these exercises were not included in courses of study or textbooks. That is to say, these writers were advocating that certain tasks be henceforth included. Exercises 112 to 116 were all added on this basis. Exercise 114 furnishes an unusual illustration. It reads: "The following drawing shows the average family's share in a city budget; how much would the average family pay in 1908?" By referring to the graph*, the reader will see that this exercise tests rather a complex ability. The pupil needs to find the figure 1908 on the horizontal scale, though the number is not shown, he needs to move vertically until he reaches the curve, then he needs to transfer to the horizontal line until it intersects the vertical axis. Note that this intersecion point it not an integral unit on the vertical axis. The point is that the pupil needs to approximate the answer and all responses were called incorrect unless this approximation was carried out. To the best knowledge of the writer, there was, at the time the inventory test was constructed, no sixth grade arithmetic course or textbook that gave either discussion or practice to the abilities tested by this exercise and yet more than one-fifth (20.4%) of the pupils tested wrote the correct result. To be sure, this is not a high number, but the reader will recall that there are some tasks in the inventory test that receive much drill and explanation in several of the intermediate grades for which the percentage of correct responses is only a little higher. Where and how did these pupils learn to do this task? Possibly in other school subjects, in general reading of newspapers and magazines, in life situations, in talking to adults, older sisters and brothers, and the like. Graphs of this type are extensively used and challenge the interest of pupils in arithmetic.

*The reader will need to refer to the test (printed by the Gazette Press, Yonkers, New York), to realize the full difficulty of this task.

It is obvious that little formal drill needs to be given to this particular type of graph.

If we keep in mind the high achievement on some of the tasks appearing in this test that receive little drill and the fact that many of the tasks securing high percentage of correct responses are treated in outside school situations, the home, the market, etc., we, as teachers of arithmetic face two depressing facts: (1) we probably drill many things that pupils would easily learn without formal instruction; (2) we seldom secure a satisfactory mastery and when we do the pupils get much of the skill and information from sources other than the formal recitations. When we do succeed, we cannot claim all the credit.

One must hasten to add that mathematics probably would not suffer by comparison with other school subjects. It is interesting to speculate what we achieve in the less well defined subjects. If our attainment is so low in a well standardized field, it is not likely that questions, in the social studies, excepting possibly one concerning the discovery of America, could be asked of three thousand beginning seventh grade pupils with anything but low returns.

8. *How Well do Pupils Achieve Objectives Concerning Which There is Agreement?*

Some elements of the inventory test undoubtedly are not worth teaching. For example, the three authors of the test were agreed that pupils need not know the answer to the following question. "Look at this: $7 - 3 = 4$. Which number is called the *subtrahend?*" It seems that few adults know the meaning of the word "subtrahend" and many other terms occurring in the terminology of arithmetic. It was not our purpose to fix this terminology, but to find out how well this was learned in the light of all the emphasis placed upon it. There are certain other elements of test that cannot be justified, certainly not at the seventh grade level. It is therefore worth while to ask "are there some elements in the test that are surely worthy objectives and how well are these learned?" The writer submitted the inventory test to 146 teachers of mathematics in his classes at Teachers' College, Columbia University, and asked them to check each item with an "A" which they felt sure pupils at the beginning of the seventh grade should know with nearly 100% accuracy; that is, allowing only an occasional slip. The following table includes

an abbreviated list of tasks which everyone of the 146 experienced teachers of mathematics accepted as valid, together with the percentage of correct responses for 3,260 pupils.

TABLE IV

5. One gallon equals how many pints? ..... (75.6)*
42. Write in figures: Fifty-nine and three hundredths ..... (76.1)
51. Look at this $5 \times 4 = 20$. Which number is called the product? .. (78 4)
56. Fifty per cent of a number is the same as what part of that number? ..... (71.4)
76. Write $\frac{3}{4}$ as a decimal ..... (74.8)
77. Write $\frac{1}{2}$ as per cent ..... (73.5)
7. One square foot equals how many square inches? ..... (60.3)
74. Write $\frac{1}{5}$ as a decimal ..... (66.6)
88. The interest on $200 for 3 years at 4% is $ ..... (60.8)
18. April has .....days; November has .....days; July has .....days? ..... (55.0)
58. % means ..... (51.4)
65. What must you do to find $\frac{2}{3}$ of $\frac{5}{7}$? Do you add? If not, what must you do? ..... (54.9)
86. What is 3% of 200? ..... (59.7)
96. Draw a figure to show that you know the meaning of the word rectangle ..... (59.8)
117. Be able to identify a check when a check is shown ..... (43.4)
29. Moving the decimal point one place to the left .....the number by 10 ..... (42.1)
32. If you have the product of two numbers, how can you find the other number? ..... (44.5)
36. What is the average of 4, 6, 8 and 10? ..... (42.7)
99. Draw a figure to show the meaning of "right angle." ..... (43.0)
40. Does 4896 ÷ 10 equal 4.896, or 48.96, or 489.6, or 4896? Draw a circle around the right number ..... (35.5)

The complete table of accepted tasks not given here included the number of elements with more than 80% correct responses. The table above shows that there were as many as twenty tasks accepted by all of the 146 experienced teachers of which not a single task received more than 80% correct responses and some dropped as low as 30%. The table shows vividly the difference between our achievement and our desires on commonly accepted objectives in the field of intermediate arithmetic.

9. *What Support Do Results from Standardized Tests Used in Surveys Give to the General Conclusions Drawn from the Inventory Test Results?*

It is very difficult for educators to realize the low degree of

*The numbers in parentheses indicate percent of correct responses.

accuracy that exists. Business men have always charged that we are turning out incompetents—persons who could not get accurate results more than four, or at most six, times out of ten when the demand is nearly ten correct results out of ten trials (that is, only an occasional slip.) Probably not even the business man is aware of the extent to which the product is deficient. In recent years survey results have been available, but their true significance seems to have been overlooked by many. For example, we read in Bonser:* "Such surveys (referring to surveys of the social usage of arithmetic) show a fair degree of proficiency in the manipulation of processes as such—a possession of the facts and processes of arithmetic." This statement should be a pleasant surprise to the business world, to the student of psychology who has been administering tests, and to the many people who have been giving achievement tests. The evidence furnished by the few studies of social usage of arithmetic now available will not suffice to support Bonser's statement. Certainly not when we can turn to results on standardized tests given in all kinds of situations to millions of children. The brief compilation that follows shows that test results support the conclusions drawn from the inventory test results. Wilson and Hoke use the following table:

TABLE OF MEDIAN PERFORMANCES FOR COURTIS ARITHMETIC TESTS, SERIES B

Grade		Addition Accuracy %	Subtraction Accuracy %	Multiplication Accuracy %	Division Accuracy %
VII	Courtis	72	85	80	91
	Courtis	67	83	78	83
	Indiana	64	84	71	84
	Iowa	70	83	79	89
	Kansas	67	83	78	87
	Boston	76	87	81	90

Here, as in numerous other places where similar results have been reported for many children, the achievement usually varies from 65% to about 75% accuracy. Wilson and Hoke also include results from the Boston Research Test. They describe the test in the following words: "The Boston Survey Test in addition of fractions consists of six simple tests of four problems each, each test having a two-minute time limit. They cover the various types of problems in the addition of fractions and they increase in difficulty from the first example in which

*Bonser: The Elem. School Curriculum, page 235.

the denominators are the same up to the last in which the common denominator can be detected with difficulty by introspection." Wilson and Hoke print the complete test, but for our purposes it will be sufficient to remember that in the easiest test the pupil is expected to add quarters and quarters, tenths and tenths, and that in the most difficult test the pupils adds sixths and eighths, twelfths and tenths, and the like. The record for grades six, seven and eight appears in the following table:

SUMMARY SHEET—CITY MEDIANS (BOSTON) ADDITION OF FRACTIONS, DECEMBER 1915

	Pupils Tested	Accuracy Median %	Accuracy Median %	Accuracy Median %	Accuracy Median %	Accuracy Median %	Accuracy Median %
VIII	1130	88.0	74.0	47.0	68.0	52.0	47.0
VII	1243	87.0	73.0	46.0	69.0	55.0	48.0
VI	1265	80.0	66.0	42.0	70.0	51.0	49.0

The easiest task gets only 88% accurate responses. In test 3, in which one denominator is a multiple of the other, (for example, the pupils add halves and sixths) the results are considerably less than 50% accurate in each of the three grades. Note particularly the very small gain from the seventh to the eighth grade in spite of the low standard of accuracy. There is in fact a drop in accuracy on three of the tests from the seventh to the eighth grade and only one per cent gain on each of the other three tests.

There is a wealth of evidence from surveys that could be used at this point. Courtis found that the answers of grade seven on his test 7 were about 57% accurate. Support also could be drawn from the measurements made by students in psychology who have used arithmetic exercises as tests of fatigue, of individual differences, etc. For the summary of the evidence the reader may turn to Thorndike, "The Psychology of Arithmetic," page 103. Consider finally the results recently secured on the Woody-McCall Form 3. This test was widely administered in the state of Michigan.* The results in the following table are for May, 1924. The report includes the

*This table was furnished to the writer by Professor Clifford Woody.

RESULTS ON THE WOODY-McCALL FORM III

Number of Problem	Grade VII Group I	II	III	All	Number of Problem	Grade VII Group I	II	III	All
1	91	84	98	93	20	75	88	85	83
2	90	85	98	93	21	61	70	71	68
3	91	85	99	94	22	76	82	76	77
4	92	89	98	95	23	57	70	68	66
5	83	92	91	89	24	52	49	56	53
6	84	94	92	91	25	76	85	82	81
7	78	83	85	83	26	57	69	66	65
8	86	89	89	88	27	47	63	63	59
9	90	83	97	92	28	37	49	43	43
10	87	92	93	91	29	51	58	62	58
11	86	96	93	92	30	18	23	34	28
12	88	97	95	94	31	20	22	25	23
13	82	85	88	86	32	21	34	34	31
14	83	93	89	88	33	17	28	14	18
15	84	92	88	88	34	9	12	14	12
16	75	86	86	83	35	14	19	24	20
17	69	77	70	71					
18	82	94	84	86					
19	68	80	79	77					
					Number of Pupils	406	369	851	1626

results from forty cities. The cities were divided into three pupil groups. Group 1 includes cities having 5,000 or fewer inhabitants. Group 2 includes cities having 5,001 to 10,000 inhabitants; and Group 3 includes cities having over 10,000 inhabitants. On the first problem:* 2+4=, 114 pupils out of 1626 pupils failed. The same number of pupils could not respond correctly to 6×2=. Nearly 200 pupils failed to answer 5×1=. Let us consider next a few questions only a little more complicated. On test 17 namely, 31—1=, nearly 30 out of every 100, or 472 out of 1626 pupils failed to respond correctly. More than this number failed on test 26: 5939×85=. On exercise 31, namely, 7/8÷6=, we find nearly 3 out of every 4, or a total of 1252 pupils out of 1626, failing to write the correct answer.

These results are recent and they represent a wide sample of the state. Woody believes that these low results are in no way due to insufficiency of time,—most pupils finish in ten minutes though the test allows twenty. Furthermore, he believes that these results are slightly higher than those gener-

*The test is not included. It is assumed that anyone reading this study up to this point is familiar with the Woody-McCall Test.

ally secured from the many cities contributing to the development of the Woody standards.

This concludes our evidence. Since our study was first undertaken Thorndike has reported much evidence on this issue. The results of the two studies are in entire agreement, as may be seen by a reading of Chapter V in Thorndike's "The Psychology of Arithmetic." The general trend of his argument appears in the following: "It is clear that numerical work as inaccurate as this has little or no commercial or industrial value. If clerks got only six answers out of ten right, as in the Courtis tests, one would need to have at least four clerks make each computation and would even then have to check many of their discrepencies by the work of still other clerks, if he wanted his accounts to show less than one error per hundred accounting units of the Courtis size. It is also clear that the habits of . . . absolute accuracy, and satisfaction in truth, a result which arithmetic is supposed to further, must be largely mythical in pupils who get right answers from only three to nine times out of ten!"

Consider also the table on the next page, from Thorndike, of which he says: "I have so computed the amount of checking a pupil will have to do to reach two agreeing numbers (out of two, or three, or four, or five, or whatever the number before he gets two that are alike), according to his mastery of the elementery processes. The facts appear in the following table:

THE EFFECT OF MASTERY OF THE ELEMENTARY FACTS OF ADDITION UPON THE LABOR REQUIRED TO SECURE TWO AGREEING ANSWERS WHEN ADDING FIVE THREE-FIGURE NUMBERS

Mastery of the Elementary Additions Times right in 1000	Approximate No. of wrong answers in sums of 5 3-place Nos. per 1000	Approximate number of agreeing answers after one checking per 1000	Approximate No. of agreeing answers after a checking of the first discrepancies	Approx. No. of checkings required over the above the first general checking of the 1000 sums) to secure 2 agreeing results.
960	700	90	216	4500
980	380	384	676	1200
990	190	656	906	470
995	95	819	975	210
996	76	854	984	165
997	54	895	992	115
998	38	925	996	80
999	19	962	999	40

"It is obvious that a pupil whose mastery of the elements is that denoted by getting them right 96 times out of 100 will

require so much time for checking that, even if he were never to use this ability for anything save a few thousand sums in addition he would do well to improve this ability before he tried to do the sums. An ability of 199 out of 200, or 995 out of 1000, seems likely to save much more time than would be taken to acquire it, and a reasonable defense could be made for requiring 996 out of 1000."

This is a high ideal in the light of the fact that not a single task of the Woody-McCall Test was done at this level by all the groups of Michigan as reported in this study and that only a very few tasks of the inventory test showed responses better than 96% accurate. We have indeed a long and hard task ahead of us as arithmetic teachers if we are to achieve the Thorndike standard.

We return now to the quotation from Bonser*; "Such surveys show a fair degree of proficiency in the manipulation of processes as such—a possession of the facts and processes of arithmetic." We must conclude that this does not seem to be supported by the data presented.

Bonser raises another important issue when he says: "But they show also a deplorable want of knowledge of how to use these processes in solving many of the important problems of daily life with economy and efficiency. The work in the schools has developed the tool without teaching its use. When we recall how largely the work in arithmetic has been taught as a thing apart and separate from the situations in life in which it is used, the result is not surprising. The remedy lies in developing the facts and processes of arithmetic as parts of projects requiring their use as tools and means of interpretation." In general his argument represents the position of those who advocate the extensive use of the project method in the teaching of mathematics. For the other side of the issue let us consider the following from Thorndike.** "There is one intelligent objection to the special practice necessary to establish arithmetical connections so fully as to give the accuracy which both utilitarian and disciplinary aims require. It may be said that the pupils in grades 3, 4, and 5, cannot appreciate the need and that consequently the work will be dull, barren, and alien, without close personal appropriation by the pupil's nature. It is true that no vehement life-purpose is directly involved by the problem of perfecting one's power to add 7 and 28

*See Bonser, "The Elementary School Curriculum," page 235.
**Thorndike, "Psychology of Arithmetic."

in grade 2, or by the problem of multiplying 253 by 8 accurately in grade 3, or by precise subtraction in long diviison in grade 4. It is also true, however, that the most humanly interesting of problems—one that the pupil attacks most wholeheartedly—will not be solved correctly unless the pupil has the necessary associative mechanisms in order; and the surer he is of them, the freer he is to think out the problem as such. Further, computation is not dull if the pupil can compute. He does not himself object to its barrenness of vital meaning, so long as the barrenness of failure is prevented. We must not forget that pupils like to learn. In teaching excessively dull individuals, who has not often observed the great interest which they display in anything that they are enabled to master? There is pathos in their joy in learning to recognize parts of speech, perform algebraic simplifications, or translate Latin sentences, and in other accomplishments equally meaningless to all their interests save the universal human interest in success and recognition. Still further, it is not very hard to show to pupils the imperative need of accuracy in scoring games, in the shop, in the store, and in the office. Finally, the argument that accurate work of this sort is alien to the pupil in these grades is still stronger against inaccurate work of the same sort. If we are to teach computation with two- and three- and four-place numbers at all, it should be taught as a reliable instrument, not as a combination of vague memories and faith. The author is ready to cut computation with numbers above 10 out of the curriculum of grades 1-6 as soon as more valuable educational instruments are offered in its place, but he is convinced that nothing in child-nature makes a large variety of inaccurate computing more interesting or educative or germane to felt needs than a smaller variety of accurate computing."

10. *What Remedial Measures Can We Take?*

Too often tests are administered in a way that does not change instruction. It is the misuse of tests that causes high school principals and superintendents to ask the question: "Isn't it time to evaluate the testing program?" It is likely that a careful study of the test results secured through widely used tests would reveal conditions that could be analyzed and remedied. In the case of the inventory test results there are at least three measures that can safely be taken in the teaching of arithmetic to grades six and seven: (1) We need to formulate in a specific way the outcomes that we desire and the

degree of mastery for which we strive. It is futile to expect to secure results in arithmetic by the "shot gun" method. It is clear that mastery of a task, unless so simple that it probably needs no formal instruction, is *not a by-product.* We must strive for it definitely. (2) We probably can still further decrease the number of outcomes in arithmetic. The number of fractions with which we expect pupils to operate has been greatly decreased during the last 20 years. Certain useless processes illustrated by the elaborate ascending and descending reduction of compound numbers found in the textbook of twenty years ago has almost vanished. No one argues that our computation skills have suffered by this movement for simplification. The issue is, *shall we give a little practice on many processess, or shall we give much practice in fixing a few things at high level?* The answer must be obvious to anyone who studies carefully the table of results on the inventory test. (3) The formal skill—a large part of arithmetic—can be driven higher by a careful application of certain principles furnished us by modern educational psychology. What these principles are, how they are applied, and the results that may be confidently expected, will be presented in Chapter VI.

CHAPTER III.

A SUMMARY OF OBJECTIVE STUDIES RELATING TO THE CHOICE OF MATERIALS IN JUNIOR HIGH SCHOOL MATHEMATICS

What the Chapter Presents. This chapter includes a brief review of the 29 objective studies relating to the mathematics of grades seven, eight, and nine. First of all we have the studies dealing with the social, business, and academic uses of mathematics. This is followed by a summary table, a sort of course of study which is accepted as the first criterion in Chapter V for selection of subject matter.

In the second place there are summaries of investigations dealing with courses of study. These constitute the second criterion of Chapter V.

Classification of Studies Dealing With Social, Business and Academic Uses.

This group of studies may be listed as follows:

(1) Analyses of pupil activities,—one study, Chase.

(2) Studies of the uses of Mathematics in general reading, six studies, Adams, Schorling (4), and Thorndike.

(3) Studies of the social and business activities of adults, nine studies, Charters, Chase, Mitchell (4), Wilson, Wise, and Woody.

(4) Investigations of the academic uses of mathematics by high school and college students in subjects other than mathematics; six studies, Callaway, Rugg and Clark (2), The National Committee, Thorndike, and Williams.

(5) Questionnaire studies, seven,—those by Jessup and Coffman, Wilson, Camerer, Moore (3), and Woody.

I. Study 1. **The Social, Business and Academic Uses as Found in Pupil Activities.*
*Chase, Sara E.***

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*Since undertaking this study Woody has published a collection of studies bearing on the arithmetic of the first six grades in the Third Yearbook of the Department of Superintendence. Some of the problems there listed have a bearing on our problem as will appear in several citations from the Third Yearbook.

**Chase, Sara E. Waste in Arithmetic, Teachers College Record, Vol. 18, September, 1917; New York, Bureau of Publications, Teachers College, Columbia University, pp. 350-371.

There appears to be only one printed study which relates to the uses of mathematics in the activities of pupils at the junior and senior high school level. Miss Chase submitted a dozen geometric items,—square, area of a triangle, and the like,—to pupils in school with the request that they were to cross out all items in the given list which they had not used in their experience out of school. It is reported that seventy-eight per cent of the children crossed out everything. It is obvious that even this study is more nearly a questionnaire than a genuine study of pupil activities. Miss Chase also tried to find the uses of mathematics in other school subjects. Teachers of manual training reported the need for board measure for computing cost.

It would seem to be desirable and feasible that this type of investigation be made, especially with respect to the use of mathematics in other school subjects. For, the school activities of a boy or girl passing through the seventh to the twelfth grade probably constitute a large fractional part of the mental activity during those years.

11. Study 2. *Uses of Mathematics in General Reading. Adams, H. W.**

In his attempt to find the mathematics used in the news, special articles, editorials, advertisements, legal notices and the like, Adams read one issue of twenty newspapers and magazines. The list of magazines was distinctly popular.

Findings.

1. The range of numbers found is suggested by such expressions as "50 words" and "250,000,000 packages of breakfast food." Most of the numbers were two-place numbers but many had nine or more places.

2. Roman numerals appeared 148 times. Most of these were below 100.

3. Amounts of United States money ranged from one cent to $100,000,000,000.

4. References to foreign money used the following: German marks, Italian lire, Japanese yen, Indian rupees, English pounds, English three-pence, English guineas, English half-crown.

5. There were 3,000 fractions. Of these, six only had a denominator larger than 16. The largest denominator was 10,000,000. Decimal fractions occurred frequently. It is important to note that there were many which did not refer to United States money. This evidence contradicts that of Wilson and others but would seem to support a

*Adams, H. W. Unpublished Masters Thesis. Department of Education, University of Chicago. See review by Franklin K. Bobbitt in Elementary School Journal, Oct., 1924. See also Woody's review of this study in the Third Yearbook of the Department of Superintendency, page 62.

common sense guess in the matter. The decimal fractions ranged from .000012 to .97. Most of the decimal fractions had less than four places.

6. Percentage was frequently used and in a great variety of ways.

7. Simple ratios were found in many different forms, for example, 30 to 1, 1-1,000,000,000, 5-5-3 ratio, 50-50.

8. Units of measure were found in many different forms. The various units of dry measure, liquid measure, linear measure, square measure, avoirdupois measure, measure of time, measure of electricity, in addition to miscellaneous measures as degrees, calorie, volt, cord, dozen, teaspoon, tablespoon, all were found.

9. The use of graphs was found to be very low.

10. There were many mathematical terms of expression illustrated by the following: "Cedar chests 40x18x18", 40% surtax, 90 miles-an-hour gale, 12 horse-power engine," and the like.

11. There was practically no reference to materials drawn from algebra, geometry and trigonometry.

In discussing the last item Bobbitt concludes that therefore it is unwise to teach the simple materials of geometry and algebra in the junior high school years as is advocated by certain leaders. How much weight should be given to this advice is suggested in the next study.

Study 3. *The Frequency of Mathematical Terms, Especially Geometric Terms in Reading Materials. Schorling.*

All mathematical terms appearing on certain pages of some magazines* were tabulated. The following table indicates the items in order of frequency. The figure following each item shows the number of times the term was found. A word was tabulated only when used in a geometric sense.

end	507	heights	103	circular	41
length	285	square	101	area	40
side	245	measures	91	opposite	38
center	203	distance	90	central	38
lines	201	plane	80	similar	36
surfaces	185	vertical	74	parallel	35
position	181	thickness	73	circle	35
point	166	ratio	62	direction	35
cylinders	149	proportion	62	arcs	32
space	130	revolved	57	divided	32
diameter	128	depth	56	equal	31
base	116	project	54	circumference	31
edge	113	horizontal	49	cone	30
sections	112	curved	46	inclined	29
angle	109	level	44	alternate	28
width	105	faces	43	dimensions	28

*The magazines were (1) Popular Mechanics every issue appearing in 1921, except the September number; (2) Popular Science every issue appearing in 1921 except the February, October and November issues.

rotates	27	elevate	6	diagonal	2
corresponding	27	feet	6	definitions	2
scale	27	hexagonal	6	elliptic	2
arms	26	incline	6	ellipses	2
right angle	26	intervals	6	encircle	2
pivot	26	middle	6	equal parts	2
contact	24	point of contact	6	extremity	2
series	22	sector	6	graduate	2
extend	20	deflection	5	graph	2
capacity	20	division	5	hemispheres	2
triangle	20	extension	5	hypothesis	2
radius	19	graduation	5	internally	2
span	19	inverted	5	intersection	2
opposite side	19	inclination	5	limits	2
shaft	18	leg	5	long	2
located	18	perpendicular	5	maximum	2
straight	16	polygon	5	minimum	2
semi-circle	15	quadrant	5	meter	2
tapered	15	rectangular	5	mid-point	2
range	15	analysis	4	polar	2
location	14	axis	4	projection	2
degrees	14	curvature	4	reverse side	2
solid	14	cubical	4	result	2
segments	14	perspective	4	ring of surface	2
succession	14	projecting	4	rotation	2
directed	13	rectangles	4	spherical	2
laterally	13	reflecting	4	spindle	2
longitude	13	semicircular	4	successive	2
altitude	12	variation	4	slanted	2
centrifugal	12	vertically	4	segmental	2
interior	12	adjacent	3	square inch	2
cylindrical	11	apex	3	taut	2
pitch	11	approximate	3	transversal	2
volume	11	coincide	3	unit of measure	2
cycles	10	coincident	3	vertical	2
grade	10	constant	3	position	2
ring	10	integral	3	wide	2
triangular	10	conclusions	3	anti-clockwise	1
alinement	9	concave	3	axial	1
alternating	9	concentric	3	approximation	1
angular	9	equi-distant	3	altitudinous	1
equivalent	9	longitudinally	3	acute angle	1
focus	9	longitudinal	3	breadth	1
latitude	9	mile	3	cubic foot	1
pyramid	9	multiple	3	concentration	1
linear	8	spirally	3	cycloidal	1
opposite end	8	terminates	3	corresponding sides	1
rotating	8	angle of inclination	2	converged	1
slope	8	coordinates	2	compass	1
conical	7	cross section	2	chord	1
elevating	7	caliber	2	clockwise	1
forms	7	diagonally	2	cylinder	1
convex	6	differences	2	depends on	1
deviation	6	directional	2	diagram	1

disproportionate	1	intersect	1	reflection	1
equilateral	1	isosceles triangle	1	reflective	1
triangle	1	lengthwise	1	rotatable	1
elongated	1	logarithmic	1	square root	1
erected	1	lengthening	1	segment of a circle	1
equitorial	1	magnitude	1	sequence	1
elevation	1	mid-way	1	straight side	1
exterior	1	mid-position	1	terminals	1
external	1	oblique	1	transversally	1
elliptical	1	octagon	1	trajectory	1
foundation	1	opposite directions	1	traversing	1
geometry	1	prisms	1	tangent	1
hemispherical	1	proposition	1	traversal	1
hypotenuse	1	parallelism	1	triangulation	1
heptagon	1	pentagon	1	T-square	1
hexagon	1	perimeter	1	unequal	1
involute	1	pi	1	whole	1
identical	1	plotted	1	zero	1
infinity	1	radiation	1	zone	1

Conclusions:

1. It is important to note the large number of geometric terms. In all there are 211 terms geometric in nature. This statement is of great importance in the light of Adams' investigation. In his study of the use of mathematics in general reading, Adams included only newspapers of the popular kind. Our study suggests that if you vary the material, keeping well within the range of what a considerable number of high school people read, a large number of geometric terms will be obtained.

2. The material in the preceding table probably has considerable stability. When approximately three-fifths of the number of pages had been tabulated, the twenty highest items were

TABLE A

The Twenty Highest Items When Approximately 3/5 of the Material is Tabulated

End	300	Center	86	Diameter	63	Width	55
Length	150	Surface	86	Height	63	Distance	49
Side	137	Point	85	Section	60	Square	49
Position	111	Edge	73	Cylinder	57	Angle	46
Lines	90	Base	68	Space	56	Thickness	43

ranked in order, as is shown in Table A. When all the material had been tabulated, the twenty highest items were again ranked, as is shown in Table B. It will be observed that only one item (thickness) is replaced by a new item (measures) when all the material is read.

TABLE B

THE TWENTY HIGHEST ITEMS WHEN ALL MATERIAL IS CONSIDERED

End	507	Surface	186	Space	128	Width	105
Length	285	Position	179	Base	116	Height	103
Side	245	Point	166	Edge	113	Square	99
Center	203	Cylinder	149	Section	112	Measure	91
Line	199	Diameter	128	Angle	109	Distance	90

3. It should not be inferred that the material constituted a random sampling of the material of this type that young people read. In fact, the terms "zero," "isosceles triangles," and "perimeter" all were found on the very last page read. No thoughtful person can observe the familiar terms appearing only once in our table without recognizing that the study needs to be extended before it is of much use as a guide. This fact does not reduce the importance of this study to zero. It is entirely adequate to show the limitation of Adams' selection of material. Moreover, it suggests the fallacy of Bobbitt's conclusion in the discussion of the study by Adams implying that high school pupils in their reading do not need to know geometric concepts.*

Study 4. *The Use of Units of Measure in General Reading. Schorling.*

The units of measure occurring on every fifth page of the following magazines were tabulated:

1. Popular Mechanics. Every issue appearing in 1921, except the September number.

2. Popular Science. Every issue appearing in 1921, except the February, October, and November numbers.

The units of measure occurring are listed in the following table. The figure following an item indicates its frequency. In order that the study might be easily checked,† the pages chosen are in all cases multiples of 5.

inch	1174	meter	19	cord	6
foot	512	ounce	18	r.p.m.	6
miles	224	cubic feet	14	square inch	6
pound	148	amperes	13	milliampere	4
tons	94	yards	12	square miles	4
degree	80	square feet	10	watt	4
gallons	59	barrel	8	gram	4
volt	59	M. Mfds.	7	acre-feet	3
H.P.	56	degrees		acreage	3
degree (F.)	46	temperature	7	cubic yard	3
acres	35	bushel	6	mileage	3

*See Elementary School Journal, October 1924

†The specified list for each page has been filed with the Bureau of Reference and Research, University of Michigan.

pint	3	francs	2	British thermo unit (B.t.u.)	1
kilogram	2	English pounds	2	calories	1
kilowatt-hours	2	sixpenny	1	candle power	1
minutes	2	span	1	rod	1
octave	2	acres	1	lineal feet	1
quart	2	teaspoonful	1	milometers	1
square yards	2	three ply	1	milograms	1
grains	2	years	1		

Conclusion:

1. Considering the amount of material included, the minimum of units of measure is very large,—there being 56.

2. The material probably has considerable stability. When approximately one-fourth of the number of pages had been tabulated, the eighteen highest items were ranked in order as is shown in Table A. When all the material had been tabulated, the eighteen highest items were ranked, as is shown in Table B. It will be seen that the four highest in Table A appear also as the four highest in Table B. Moreover, only one item in Table A does not appear in Table B.

TABLE A

The Eighteen Highest Items When Approximately ¼ of the Material Had Been Tabulated

inch	263	H.P.	19	cubic feet	7
foot	184	gallons	17	yards	6
miles	62	tons	16	square feet	4
pound	52	ounce	10	amperes	3
volt	33	meter	9	bushel	3
degree	21	degree (F)	7	acres	2

TABLE B

The Eighteen Highest Items When All Material Had Been Considered

inch	1174	gallons	59	ounce	18
foot	512	volt	59	cubic feet	14
miles	224	H.P.	56	amperes	13
pound	148	degree (F)	46	yards	12
tons	94	acres	35	square feet	10
degree	80	meter	19	barrel	8

3. In spite of the stability indicated in the preceding section, the reader must not be led into thinking that the whole issue concerning units of measure has been determined. The sampling was not sufficiently extensive, in fact the term "rod" appeared in the very last page read.

Study 5. *Frequency of Units of Measure in the Reading of Newspapers.—Schorling.*

The following table shows the frequency of certain units of measure appearing in a single issue of each of ten newspapers. This was an effort to find out the order resulting by varying the material slightly from that listed in the preceding study. Some of the papers listed were Sunday editions. In each case the date of the issue is indicated; thus for the New York Times it is February 15, 1922.

THE FREQUENCY OF UNITS OF MEASURE IN THE READING OF NEWSPAPERS

	N. Y. Times	Boston Evening Transcript	Chicago Daily Tribune	San Francisco Chronicle	Kansas City Star	San Antonio Express	Miami Herald	Oregon Journal	Times Picayune	Indianapolis Star	Total Frequency
	2/15 22	2/21 22	2/23 22	2/17 22	2/20 22	2/23 22	3/ 4 22	3/3 22	2/21 22	2/23 22	
Foot	30	62	68	72	55	58	45	23	59	11	483
Mile	51	30	35	124	38	34	47	24	59	30	472
Acre	3	14	34	11	46	39	60	40	15	29	321
Inch	37	36	19	16	11	14	20	36	24	9	222
Barrel	23	16	5	26	8	6	4	3	56	12	159
Yard	24	22	12	11	4	8	26	9	14	4	134
Sq. Ft.	50	7	54	2	7	1	....	....	....	1	122
Bushel	....	10	4	4	42	....	....	6	7	4	77
Gallon	3	3	1	13	1	1	4	21	19	1	69
Furlong	15	....	....	17	....	....	....	....	10	....	42
Quart	3	....	1	....	1	....	....	3	8	7	23
Cu. Yd.	....	....	....	....	....	21	....	1	....	....	22
Cord	....	....	....	....	....	....	....	11	4	....	15
Pint	1	....	....	....	....	....	....	7	4	2	14
Meter	....	....	....	....	2	....	....	....	5	....	7
Cu. Ft.	....	1	....	....	....	....	1	1	2	....	5
Sq. Yd.	1	....	....	....	....	....	....	....	....	....	3
Sq. In.	1	....	....	....	....	....	....	....	....	....	1

Findings:

1. There are six units of measure that appear in all papers and eleven that appear in a considerable number.

2. It is of special interest to note that the term "furlong" appears with considerable frequency in newspapers published in the coast cities.

3. It is obvious that the study needs to be extended for the frequency of certain units probably frequently found in common life is low.

Study 6. *The Frequency of "Uncommon" Fractions—Schorling.*

The available investigations present clear evidence that the world's computation is done with relatively few denominators of fractions. In fact, fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 16 probably cover more than ninety-nine per cent of the computation in ordinary life. The next study attempted to find out whether fractions with denominators other than these were encountered in general newspaper or magazine reading.

The table which appears on the following pages lists the eighteen newspapers and twelve magazines for which the material was tabulated. The material was more extensive than appears because some of the issues were Sunday papers. The table shows the frequency of each "uncommon" fraction, together with a suggestion of the setting in which each appeared.

Paper read and date	Fraction Found	Frequency	Subject Discussed
New York Globe, Feb. 10, 1922	1/32	1	Hair Clippers
	1/32, 3/32	2 each	Bond Sales
	5/32	1	Bond Sales
	81/100, 56/100	1	Legal Notice
New York Tribune, Sunday, Feb. 12, 1922	1/50, 1/40	1	German Indemnity
	1/19		
New York Times, Feb. 14, 1922	31/35, 14/26	3 each	Billiard Scores
	1/23, 14/26,		
	1/22, 16/39, 23/39	1	Billiard Scores
	1/32, 3/32	2 each	Short term Gov. loans
	31/32, 5/32, 9/32, 25/32, 13/32	1	Short term Gov. loans
New York Evening Mail, Feb. 15, 1922	1/7	1	Book Making
New York Sun, Feb. 16. 1922	46/100, 53/100	1	Diamond Weights
	14/26, 45/51	1	Billiard Scores

Paper read and date	Fraction Found	Frequency	Subject Discussed
New York Evening Mail, Feb. 17, 1922	11/17	1	Billiard Averages
	1/22, 3/26		
	1/23	3	Billiard Averages
New York Sun, Feb. 17, 1922	30/100	1	Legal Notice
Kansas City Star, Feb. 20, 1922	None		
San Francisco Chronicle, Feb. 30, 1922	80/100	1	Resolutions of Sale
	3/32	3	Sugar Market
	1/32	2	Sugar Market
Boston Evening Transcript, Feb. 21, 1922	47/100, 71/100	1	Mortgage Sale
	48/100	2	Mortgage Sale
Indianapolis Star, Feb. 23, 1922	None		
New Orleans Times-Picayune, Feb. 23, 1922	50/100	2	Diamond Size
Chicago Daily Tribune, Feb. 23, 1922	None		
San Antonio Express, Feb. 23, 1922	25/32	1	Mexican Oil Exchange
Miami Herald, Mar. 4, 1922	1/64	1	Bottle Making Machine
Industrial Digest, Dec. 24, 1922	1/21	1	Excess in Chocolate
Industrial Digest, Dec. 24, 1922	3/64	1	Auto Service
	1/40	1	Hours to Minutes
	1/7	1	Welding
Industrial Digest, Oct. 1921	1/000000	1	Magnetion

By studying the preceding table, we note that, in this material, estimated at 2,552,000 words, we find only 72 fractions with "uncommon" denominators. Of these fractions 16 are used in such situations as scores in horse racing and billiards.

It is sufficiently close to the fact when we say that the finding of a fraction with a denominator not in our list is as difficult as the proverbial needle in the hay stack.

Study 7. *An Inventory of the Encyclopedia Britannica.—Thorndike.*

This study represents Thorndike's effort to see "how far algebra has penetrated into the intellectual and practical life of men of intelligence, an achievement comparable to that of high school graduates." Thorndike examined the first two hundred pages of each volume from I to XXVIII, and has noted all references to mathematics beyond arithmetic.

Findings:

1. The articles using mathematics beyond arithmetic in any form are 3.57 per cent of the total number of articles, but they occupy 18.55 per cent of the total space.

2. Articles on the shapes of buildings, land, etc., include such terms as rectangular, rhombic, quadrilateral, conical, cylindrical, triangular, trapezoidal, spherical, square, octagonal, hexagonal, quadrangle, as these are applied to grounds, towers, tunnels, and the like.

3. "For utilitarian values in the first year of algebra one must pay much attention to the formula and to the statistical graph."

4. "The student whose aptitudes and capacities will probably lead him into any one of a large number of fields of advanced study as physics, engineering, psychology, chemistry, education, electricity, economics, aeronautics, social sciences, ballistics, navigation, etc., should have preparatory work, not merely in algebra but in many other branches of mathematics to which rigorous algebra is the necessary antecedent and tool."

5. "Very few, if any, cases appeared in all the textbooks examined or in the 5000 pages of the Britannica where ability to factor $a^3—b^3$ or a^3+b^3 was demanded. There were few, if any, cases where the ability to factor $a^2—b^2$ was demanded. Algebraic abilities form a series of diminishing utility, beginning with reading formulas and graphs or the relation of one variable to another, and ending with short-cut methods of division by knowledge of special identities."

6. "The mere knowledge of the language of algebra has more utility than educators have thought, while skill in computing has less. Educators are prone to think it folly to learn algebraic symbols at all if we don't compute with them. They forget that we have to read them, and perhaps twice for every once that we compute with them. The knowledge of and ability to use formulas in solving quadratics, answering problems about progressions, and determining coefficients and exponents in the binomial expression, which figure so largely in school examinations, are toward the low end for utility."

7. "We find in the case of arithmetic that people often do not use it when it would be to their advantage to do so, because they have not

*Thorndike, "The Psychology of Algebra," Chapter on Uses of Algebra.

mastered it. Writers may use long and clumsy verbal descriptions because they lack the ability and habit of framing a formula to describe a state of affairs or relation. Very little use of algebra is made by accountants, but this may be due in part to traditions from the book-keeping of the small shop whence scientific accounting developed, rather than to essential needs and desirabilities."

8. "The following* seems worth requiring of all students in Grade 9 if anything is worth requiring of them at all, and if they have not learned its content previously in arithmetic."

The notion of symbolism, ability to "let...=the number of..."

"Ability to read simple formulas, including knowledge of positive integral exponents, of a^2 or $a^{1/2}$, a^3 or $a^{1/3}$, or single parenthesis, and of the ommission of the $\times$ sign and of 1 as coefficient."

"Ability to evaluate and solve such formulas, including the interpretation of negative numbers as answers, when the unknown forms one member of the equation."

"Ability to read graphs of the relation of one variable to another in the+ + quadrant, when the nature of each variable is understood."

III. *The Social and Business Activities of Adults.*

Under this heading we shall review nine studies,—those by Charters, Chase, Mitchell (4), Wilson, Wise, and Woody. The material analyzed is shown in the following table:

*Author***	*Basis of Study*
Charters	7337 charge checks and 4304 cash checks collected in a department store.
Chase	Test given to adult students at Teachers' College.
Mitchell	Standard cook book, payroll, mark-down sales, and hardware catalogue.
Wilson	14,583 problems presumably solved in the daily lives of 4,068 persons in 155 occupations.
Wise	7345 problems presumably solved in the daily lives of persons in 148 occupations.
Woody	4661 bills of sale collected from hardware, grocery, and department stores.

Study 8. *Charters.† Uses of Mathematics by Salespeople in Department Stores.*

To discover exactly what operations are important to salespeople the sales checks were analyzed and the processes tabulated.

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*Probably the most important summary in print bearing on the Algebra phase of our problem.

**This outline was suggested by Professor Clifford Woody.

†Charters, W. W.—"Curriculum Construction," pp. 231-236.

Findings:

1. The maximum number of addends was 18; there were few problems with more than 3. Five places constituted the maximum.

2. Salespeople add in making change, that is if the purchase amounts to 43 cents and a dollar is given in payment, the salesperson counts forward something like this: 43, 44, 45, 50, 100.

3. In multiplication 97% of the cases fell within the multiplication tables to 12. There were eight cases of multiplicands with four places.

4. Divisions are used in the form of multiplying by fractions.

5. In both types of checks there were 1178 cases of common fractions; denominators with 2 constituted 56% of the cases; 4 were used in 25% of the cases and 99% of the fractions had denominators 2, 3, 4, 5, 6, 8, 10, 12 or 16.

6. Decimal fractions were used to a very limited extent.

7. The conventional short cut of dividing by 10 was used though apparently not recognized as such by Charters.

Study 9. *Chase. Use of Geometry by Adults.*

A test was given to 140 men and women engaged in different industries. Those taking the test were asked to cross out all items in the given list which they had not used in their experience out of school.

The following table gives the percentages of men and women using the different items listed:

Area of	Men	Women	Entire Surface of	Men	Women	Volume of	Men	Women
Square	66⅔	27	Cube	28	8	Cube	39	9
Rectangle	61	26	Prism	20	5	Prism	18	8
Triangle	42	8	Pyramid	9	0	Cylinder	36	9
Circle	32	12	Cylinder	24	5	Square	41	12
Circle	32	12	Cylinder	24	5	Square root of number	41	12

*Studies 10-13—Mitchell**

The frequency of elements of the course of study in arithmetic and the manner of their use (a) in a standard cook book; (b) in the payrolls of a number of artificial flower and feather factories; (c) in marked-down sales advertisements; and (d) in a general hardware catalogue.

*Mitchell, H. Edwin. "Some Social Demands on the Course of Study in Arithmetic," Seventeenth Yearbook of the National Society for the Study of Education, Part I. Bloomington, Illinois, Public School Pub. Co., 1918, pp. 7-17. See also Woody's summary of this study in the Third Yearbook, Department of Superintendence, page 49.

Findings:

Study 10. *The Cook Book Study.*

The most frequently used fractions had denominators of 2, 3, 4 and 8.

Study 11. *From the Factory Payrolls.*

1. The word "dozen" and fractional parts of twelve were frequently used. The commodities were made and sold by the dozen.
2. To compute earnings the arithmetical processes occurring were: addition of all possible fractions with 12 as a denominator to integers and to mixed numbers; multiplication by a fractional, integral, or mixed number of dozens; addition of United States money.
3. The fractions most frequently used were twelfths, fourths, thirds, and sixths.

Study 12. *From Marked-down Sales Advertisements.*

Discount notes were ordinarily expressed as per cents, the exceptions being halves and thirds. The most common practices were to use 10 per cent, 20 per cent, 25 per cent, one-third and one-half off.

Study 13. *From the General Hardware Catalogue.*

1. Many articles were sold by the dozen, hence the frequency of fractions and mixed numbers involving 12 as the denominator of a fraction.
2. Fractions with denominators of 16, 32, and 64 occurred. These seldom involved computation but the dealer had to read and understand them.
3. Quantities listed by avoirdupois weight were descriptive but addition of weights was necessary in determining freight rates.
4. The units of measure occurring most frequently were: inches, feet, yards, ounces, pounds, dozen, gross, quarts.
5. The common semi-mathematical phrases often used in selling were: per each article, per each dozen, per gross, per hundred, per foot, per hundred feet, per square foot, per hundred square feet.

Study 14. *Wilson*—A Collection of Problems in Arithmetic*

With the co-operation of superintendents and teachers in schools of the eastern Middle West, Wilson collected 14,583 problems from 4,068 different people, representing 155 different occupations. The teachers asked the children in grades six, seven and eight to get from their parents each night, for a period of two weeks, a statement of the uses which they had made of arithmetic during that particular day. The problems thus collected constitute the data used in Wilson's study. Wilson assumes that the data constitute a random sample of arithmetic problems met in common, everyday life. This is a study to which frequent reference is made. The summary of conclusions which follows and also for the study of Wise which follows is, with only slight modifications, that given by Woody in the Third Yearbook:

*Wilson, G. M. "The Social and Business Uses of Arithmetic." Teachers College Contributions, No. 100. New York, Teachers College, Columbia University. pp. 1-62.

Findings:

1. Eighty-five per cent of all problems involved the use of money in either the buying or selling of goods. Labor and wages, interest, rent, and insurance covered two-thirds of the problems involving money, but not involving buying or selling. Problems not involving money involved hours of labor, measurement of capacity of bins, cisterns, cribs, tanks, estimates of building materials, etc.

2. Of all the problems involving buying and selling 45.9 per cent related to food; 17.9 per cent to clothing. For all occupations the ten topics involved the greatest number of times in the different problems were: groceries, drygoods, labor, milk, making change, meat, eggs, clothing, butter and fuel.

3. Multiplication, addition, subtraction, division, and fractions constituted 90.6 per cent of all the problems. Very little use was made of proportion, decimals, apothecaries' weight, square root, partial payments, and troy weights. When these latter topics were involved the problems were too involved for the elementary grades.

4. In the problems involving addition only 2.5 per cent of the problems had more than four places in the largest addend, and almost all of the problems had either one or two place numbers in the multiplier. In division, 39.6 per cent of all the problems had one number in the divisor; 93.4 per cent had one or two place numbers.

5. The most commonly used fractions had denominators of halves, thirds, fourths, fifths, eighths, and twelfths.

Study 15. *Wise*—A Collection of Problems Solved by Adults.*

Wise's study is similar to the preceding. The teachers of grades six, seven, and eight asked their pupils to bring from home on each of five days, at least one problem encountered by the parents in the day's work. These problems were collected through the co-operation of school people in the States of California, Illinois, Iowa, Minnesota, Texas, and Wisconsin. Wise analyzed 7.345 different problems collected from men and women working in 148 different occupations. Professor Daniel Starch of the University of Wisconsin directed the study.

Findings:

1. There was little difference in type between the problems collected from the city and those from the rural districts; nor was there any noticeable difference in type between the problems collected from one section of the country and those collected from another.

The fractions occurring most frequently were: ½, ⅓, ⅔, ¼, ¾, 1/16, and ⅛. Most of these were used in multiplication and make up 93.9% of all fractions found. Problems in which common weights and measures were used occurred frequently. Few problems in bank discount, compound interest, compound proportion, insurance, plastering, painting, masonry, were found.

*Wise, Carl T., "A Survey of Arithmetical Problems Arising in Various Occupations." Elementary School Journal, Vol. 20, No. 2, October, 1919. pp. 118-136.

3. No problems were received involving taxes, stocks or bonds, the metric system, etc., a matter that should have given both Wise and Wilson some concern.

Study 16. *Woody*—The Arithmetic Needed by Clerks.*

Woody tried to find out the arithmetic needed by clerks in selling goods and by the public in buying their goods. He analyzed 4,461 bills of sale. These were obtained from three large stores of Seattle—a wholesale and retail hardware, a wholesale and retail grocer, and a large department store. Woody tried to supplement his objective study by interviews with managers concerning needs for arithmetic. Woody summarized his findings as follows:

Findings:

1. Nine hundred forty-five of the 4661 bills of sale required no computation whatsoever, other than in the payment of the bills.

2. There was no use of decimals save in connection with United States money.

3. Of the problems in addition encountered none had more than five places in the addend, and few had more than six addends. The most common type of problem in addition consisted of adding two three-place numbers.

4. Few problems in subtraction, other than making change, were encountered. In the grocery store only one of the 206 bills of sale involved any subtraction other than making change. When subtraction was involved, usually the problems had but three or four numbers in the minuend.

5. In multiplication, the vast majority of the problems had but two or three-place numbers in the multiplicand and one or two-place numbers in the multiplier.

6. By far, the most common problems in division had two or three-place numbers in either the dividend or the divisor. Division as a process was used less than addition or multiplication.

7. The most commonly used fractions had denominators of 2, 4, 6, and 12, although 3, 5, 9, 10, 16, 24 and 144 were used to some extent. The most common units of sale determined very largely the fractions used. Dozens or gross were often employed and thus the fractions with denominators of 12 and 144 were used. Such fractions were rarely reduced to lowest terms. There was much multiplication of a two, three or four-place number by a simple fraction, or by a mixed number having one, two, or three places and a fraction. There was little use of addition, subtraction and division of fractions.

8. Denominate numbers were encountered as units of sale, but no reduction into larger or smaller units took place.

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*Woody, Clifford, "Types of Arithmetic Needed in Certain Types of Salesmanship." Elementary School Journal, Vol. 22, No. 7, March, 1922, pp. 505-521. See also Third Yearbook, p. 52.

9. Discount was widely used, especially in the hardware store. The most common rates of discount were 5, 10, 20, 25, 30, 35, 40 and 50 per cent. Inquiry revealed that discount tables were used and that little calculation was employed.

10. Interviews with the managers of these stores to supplement the analysis of the bills of sale revealed the following significant points: points:

(a) The goods were marked in terms of the decimal system to make calculations easy.

(b) Goods were sold by convenient units or groups of units, and were not usually sold by using a simple unit of a multiple of the total cost of a large amount of goods.

(c) Measuring devices, weighing devices, calculating machines. calculating devices, such as discount books, price tables, cash registers, central cashiers, etc., have reduced to a minimum the amount of arithmetic used by clerks. Often times the manipulating of a mechanical device and reading numbers was all the arithmetic needed by the clerks in selling goods. If there was no central cashier, in addition to the ability to read numbers, the clerks must be able to make change.

11. While the evidence indicated that the clerks under consideration made little use of arithmetic, the author argued that the consuming public needed arithmetic for its own satisfaction in checking the accuracy with which the calculating and measuring devices were manipulated. He argued for much emphasis on mental arithmetic, i. e., calculating without pencil or paper, and for much emphasis on quickly estimating or approximating a correct answer to the problem involved.

IV. The Academic Uses of Mathematics in High School and College Studies.

In this section five studies will be reviewed—those by Gallaway, Rugg and Clark, The National Committee, Thorndike, and Williams.

Study 17. Gallaway—The Mathematics of an Introductory Course in Clothing.*

Gallaway, in trying to determine what mathematics is needed, to do satisfactory work in a freshman college course in clothing, analyzed (1923) Baldt's "Clothes for Women" and recorded all the mathematics found and listed the nature of the different operations needed.

Findings:

1. Gallaway found that 111 mathematical concepts were necessary for intelligent reading of the text. There were 43 geometrical terms used for the most part in mensuration. The remaining terms were arithmetical; of these, line, equal, center, measure, width, length, and point occurred most frequently.

*Gallaway, Mrs. T. T., "Mathematics Needed in a Freshman Course in Clothing." Unpublished study reported in Charter's "Curriculum Construction." pp. 241-43.

2. The most common units of measure were: inch, dollar, yard, head, and year.
3. The fractions most common were: halves, quarters, sixths, eighths, tenths, twelfths, and sixteenths. There were 12 cases of decimal fractions.
4. Most of the whole numbers found were expressed in less than four digits and none of them had as many as five digits.
5. There were 77 Roman numerals, but none with more than two places.
6. The textbook examined had five chapters on the drafting and use of patterns. These were the chapters that frequently employed "rhetorical algebraic equations." Gallaway describes these equations as follows: "These equations represent arithmetical problems to be solved, geometric constructions to be made, or more frequently, a combination of the two." The flowing is an example of the kind of equation described: "A. B. equals one-half hip measure minus one-eighth of one-half width around bottom of the skirt." It was reported that 430 equations of the sample given above were used. The solution of these equations required the following operations and processes:
   (1) Translating the rhetorical algebraic equation into a symbolic numeral equation.
   (2) Multiplying a whole or mixed number by a fraction.
   (3) Subtracting a whole or a mixed number from a whole or mixed number.
   (4) Adding a fraction to a whole or mixed number.
   (5) Finding a fraction of a fraction.
   (6) Reducing a mixed number to an improper fraction.
   (7) Reducing an improper fraction to a whole or mixed number.
   (8) Reducing yards to inches.
   (9) Drawing a line of a given length.
   (10) Reducing inches to yards.
   (11) Division of whole numbers, mixed numbers, or fractions.
   (12) Drawing a line parallel to a given line.
   (13) Drawing a line perpendicular to a given line.
   (14) Drawing a circle or the arc of a circle, having given the center and radius.
   (15) Reading geometrical figures by means of letters representing points.
7. Percentage was used in two chapters and nowhere did this require anything more than an understanding of the terms "percentage" and the ability to find a certain per cent of a given sum of money.

Study 18. Rugg and Clark—Mathematics Needed for Certain School Texts.*

Rugg and Clark reported (1918) two studies. The first attempted to find the use of ninth grade algebra in other school subjects—plane geometry, solid geometry, physics, and chemistry. The second study inventories the algebraic material in trade school texts and corporation school material.

*See Reconstruction of Ninth Grade Mathematics.

1. For their first study they checked the percentage of the total number of problems that employed any one of 33 algebraic topics. Their study is not sufficiently detailed to be of great help on our problem. It was found that in plane and solid geometry the following topics occur frequently: evaluation, equations of the first degree and one unknown, multiplication and division (for our study we really should know what *kinds* of division problems), radicals, removal of parentheses, exponents, addition and subtraction, equations of the second degree and one unknown, multiplication and division of fractions, fractional equations, and ratio and proportion. Frequently no use was made of factoring and partial products.
2. When we come to the subject of physics, we find that evaluation, equations of the first degree and one unknown, multiplication and division, ratio and proportion, addition and subtraction of fractions, multiplication and division of fractions, are of importance in the order named.
3. In chemistry the order is as follows: ratio and proportion, equations of the first degree and one unknown, multiplication and division, evaluation. Another way of emphasizing the findings is to say: "For 18 of the 33 topics now taught in first year algebra there is absolutely no call. Similarly chemistry has no need for 25 of them and plane geometry for 17 of them."
4. The investigators found a high percentage under H.C.F., L.C.M., and L.C.D., but these seldom exceeded in difficulty the ability to clear a fractional equation in which there was a single numerical denominator.

Study 19. Rugg and Clark—The Material Used in Trade School Texts and Corporation School Texts.*

Findings:

1. The geometry was very simple; it was of the type that could be given in the seventh and eighth grades. There was practically no demonstration.
2. Trigonometry came in for definite recognition.
3. The algebra was very simple, in multiplication nothing more complicated was found than the multiplication of two binomials.
4. Only three of the trade school texts included factoring. The topic "special products" was included in but two corporation school books and in only one trade school book.
5. But little use was made of graphs. The authors asserted that at this time the corporation school "did not see clearly the use of graphs as a fundamental method of representing numbers."
6. Ratio and proportion was found in all texts.
7. Evaluation was found to be the most frequently used operation.
8. Radicals and quadratics were found in only one book.
9. The The verbal material emphasized evaluation, simple equations, ratio and proportion, the literal equation, and the literal formula.
10. There was no attention paid to special product, factoring, fractions, fractional qutaions, H.C.F. and H.C.M.

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*See Pages 45 and 46 of the "Reorganization of Mathematics in Secondary Education."

Study 20. National Committee on Mathematical Requirements —What Mathematics is Needed for Certain College Courses?

The purpose of this investigation was to secure information as to the content of high school courses of instruction most desirable from the point of view of preparing to carry courses in college successfully. The members of the Committee asked a number of college teachers, prominent in their field, to assign to each of the topics of the following table an estimate of its value as to preparation for the elementary courses in their respective subjects, according to the following scale: "E" to mean essential, "C" of considerable value, "S" of some value, and "O" of little or no value.

The following table is a combination of two tables that appear in the report of the Commitee.* The table gives a summary of the results for the Physical Sciences (Astronomy, Physics, and Chemistry), and for the Social Sciences (History, Economics, Sociology, and Political Science.) The figures in the "N" column give the number of replies received. The figures in the first four columns under each group are percentages of the number of replies received. The figures in the second-last column headed "E" are taken from the preceding columns, choosing in each case the higher of the two "E" ratings there given. The last column, headed "E+C", gives in each case the sum of the two ratings for "E" and "C", divided by 2. This is not strictly an average, but a number arbitrarily chosen. Note also that when a number in the last two columns does not exceed 60 it is rejected.

The array of topics, in the order of importance, as determined by the numbers in the last column, is as follows:

1. Simple formulas—their meaning and use.
2. Ratio and proportion.
3. Numerical computation with approximate data—rational use of significant figures.
4. Graphic representation of statistical data.
5. The linear function, $y = mx + b$.
6. Negative numbers — their meaning and use.
7. Mathematical and empirical graphs as a method of representing dependence.

*See page 45 of the final report.

	Physical sciences.					Social sciences						
	E.	C.	S.	Q.	N.	E.	C.	S.	O.	N.	E.	
Negative numbers—their meaning and use	79	5	10	5	39	45	17	22	17	18	84	73
Imaginary numbers—their meaning and use	23	21	25	31	39	13	13	37	37	16	...	..
Simple formulas—their meaning and use	93	5	2	..	41	47	26	21	5	19	98	85
Graphic representation of statistical data	57	25	15	3	40	57	24	14	5	21	82	81
Graphs (mathematical and empirical):												
(*a*) As a method of representing dependence	62	16	22	..	37	15	54	15	15	13	78	73
(*b*) As a method of solving problems	45	20	28	6	25	18	18	46	18	11	65	..
The linear function, $y=mx+b$	78	14	8	..	37	29	29	14	29	14	92	75
The quadratic function, $y=ax^2+bx+c$	59	21	17	3	34	8	8	33	50	12	80	..
Equations: Problems leading to—												
Linear equations in one unknown	98	2	..	..	41	40	7	20	33	15	100	73
Quadratic equations in one unknown	78	15	5	2	40	31	8	8	54	13	93	66
Simultaneous linear equations in 2 unknowns	71	24	3	3	38	33	8	..	58	12	95	68
Simultaneous linear equations in more than 2 unknowns	43	29	23	6	35	8	8	17	67	12	72	..
One quadratic and one linear equation in 2 unknowns	40	24	27	9	33	..	9	9	82	11	64	..
Two quadratic equations in 2 unknowns	31	19	28	22	32	..	9	..	91	11	...	..
Equations of higher degree than the second	10	32	32	26	31	..	..	9	91	11	...	..
Literal equations (other than formulas)	43	18	32	7	28	..	10	40	50	10	61	..
Ratio and proportion	84	8	3	5	39	37	26	32	5	19	92	82
Variation	50	13	20	17	30	17	33	25	25	12	63	..
Numerical computation:												
With approximate data—rational use of significant figures	61	36	..	3	39	40	27	20	13	12	97	82
Short-cut methods	27	38	24	10	37	29	35	23	12	17	65	64
Use of logarithms	62	29	7	2	42	12	29	29	29	17	91	66
Use of other tables to facilitate computation	24	45	26	5	38	18	29	41	12	17	69	..
Use of slide-rule	24	39	26	12	38	11	39	28	22	18	63	..
Theory of exponents	36	31	25	8	36	..	21	21	57	14	67	..
Theory of logarithms	34	26	21	18	38	7	13	20	60	15	60	..
Arithmetic progression	16	32	38	13	37	23	29	12	35	17	...	..
Geometric progression	19	27	40	14	37	23	25	18	35	17	...	..
Binomial theorem	35	32	18	13	37	13	20	27	40	15	67	..
Probability	9	32	41	19	32	20	35	35	10	20	...	..
Statistics:												
Meaning and use of elementary concepts	23	28	31	17	29	55	36	5	5	22	91	71
Frequency distributions and frequency curves	15	19	35	32	26	47	33	10	10	21	80	..
Correlation	11	18	39	32	28	33	47	14	5	21	80	..
Numerical trigonometry:												
Use of sine, cosine, and tangent in the solution of simple problems involving right triangles	68	21	3	8	38	..	..	25	75	12	89	64
Demonstrative geometry	68	15	12	6	34	..	21	43	36	14	83	..
Plane trigonometry (usual course)	57	27	11	5	37	8	23	31	38	13	84	..
Analytic geometry:												
Fundamental conceptions and methods in the plane	32	45	19	3	31	..	15	38	46	13	77	..
Systematic treatment of—												
Straight line	34	37	20	9	35	9	9	18	64	11	71	..
Circle	29	43	20	9	35	..	18	9	73	11	72	..
Conic sections	18	41	26	15	34	..	9	18	73	11	...	..
Polar coordinates	18	26	41	15	34	..	..	18	82	11	...	..
Empirical curves and fitting curves to observations	12	38	38	12	34	8	..	25	67	12	...	..

8. Linear equations in one unknown.
9. Statistics—the meaning and use of elementary concepts.
10. Simultaneous linear equations in two unknowns.
11. Quadratic equations in one unknown.
12. The use of logarithms in computation.
13. Short-cut methods of computation.
14. Numerical trigonometry—the use of the sine, cosine, and the tangent in the solution of simple problems involving right triangles.

The preceding order of topics does not appear in the Report of the National Committee on Mathematical Requirements, nor does it appear in Thorndike although in each case there is clearly an effort to determine the order of importance of the topics through the manipulation of the data in the first ten columns of the preceding table. It is the opinion of the writer that the *foregoing array is very significant.*

Study 21. Thorndike—Mathematics Needed in Certain High School Texts.*

Thorndike and his associates examined from two to four high school textbooks in each of the following groups: Social Sciences, Physical and Biological Sciences, and Practical Arts. Altogether forty-four books were analyzed in their effort to find out what algebraic abilities are useful and to see how great the need for these abilities is. The study listed under the following heads the kind of algebra needed:

(1) Manipulation of complicated polynomials.
(2) Formation and solution of identities and equations.
(3) Formation and evaluation of formulas.
(4) Development and use of the notion of function.
(5) Construction and interpretation of graphs for:
  (a) statistics
  (b) functions.

"Findings:"

At the end of a table which includes their data, the investigators state the following conclusions:

1. Omitting from consideration courses in mathematics itself, there is no need in high school studies for facility in complex manipulation of polynomials.
2. In present textbooks there is no use made of the mathematical conception of function.

*"Psychology of Algebra," page 70.

3. Except in chemistry and physics and agriculture the study of equations has at present no utilization in high school work.
4. The making of formulas is practically not required of the high school student.
5. The comprehension and evaluation of formulas is required only in physics, chemistry, and in the physical and chemical parts of general science.
6. The mathematical graph, either as illustration of the scope of the formula, or as vivifying the concept of function, practically does not occur in any high school work.*

In addition to the preceding formal summary, the following sentences in Thorndike's discussion have a bearing on our problem:

"As a preparation for high school subjects outside the mathematics courses as they are at present taught, so far as may be judged from textbooks, the most extensive need for algebra is in the reading of statistical graphs, and to a far lesser degree, in the making of them."

"As a preparation for physics the chief value lies in the mastery of the formula, particularly in handling the transformation known as 'changing the subject of the formula.' The student unacquainted with algebra is entirely at a loss in proceeding with this type of work."†

"The solution of physics problems demands the ability to choose the appropriate formula, to make substitutions, and to solve the resulting equations. This involves skill in clearing of fractions (chiefly in proportions), transposing, combining and taking numerical square root."

"For the solution of chemsitry problems, the ability to form a correct proportion is of high value. The algebra problems which can be applied with advantage in this field are those dealing with proportions, mixtures, and alloys."

"The next question that presents itself is: Can algebra be utilized by other high school subjects, and with reinforced vitality in the algebra itself?"

"By the formation and solution of equations certain science courses might be much enriched. However, the need would seem to be for extreme facility in the handling of easy equations rather than the formation and solution of complicated ones. Especially is this true of the ordinary proportion equation. Charles' Law, Boyle's Law, the laws of the five simple machines (usually given in first-year general science as well as in physics and chemistry); changing recipes in cooking; adjusting patterns in sewing; obtaining lengths for use in mechanical drawing, or in scientific free-hand drawing; plan-making and model-making in wood and metal work and other shop work; all these demand ready use of the proportion."

"The utilization of the statistical or pictorial graph has already become general in all high school subjects. What is needed is the development of the critical faculty in reading and in evaluating such graphs."

"Teachers of many subjects would find it advantageous if they could utilize the mathematical graph—that is, the graphic presentation of a

*Its only utilization in high school physics lies in the reading and construction of graphs illustrating the resolution of forces.

†Nunn, "The Teaching of Algebra," 1914, p. 78, also pp. 14, 15.

law or a function. It is the aim of science to formulate is conclusions not as mere statistics, but as mathematical laws."

"Involved manipulation of polynomial expressions is not a justifiable way of using the high school student's time."*

"Since the application of equations in other high school subjects is chiefly in the proportion form, mastery of that form and other easy equation forms should be secured."

"It would be profitable to extend the field of application of the construction of formulas as well as their evaluation."

"There is need for the careful development of the art of criticism as applied to graphs."

"The presentation of laws by means of mathematical graphs should be encouraged."

"The function concept should be used when advantageous, but with economy.

Study 22.—Williams†—The Mathematics of Freshmen Chemistry.

This is an effort to list the mathematics necessary to carry successfully a course in freshman chemistry in which the text book was Noyes' "A Textbook in Chemistry." The operations and expressions found in both the expository material and the problems were listed.

Findings:

1. Williams found that 124 mathematical concepts were used 1,156 times in the body of the text. Of these, per cent and volume constituted about half the cases.
2. There were 73 denominate numbers used 1,937 times; of these, degree, gross, and liter were most frequent.
3. In the body of the text 55 different fractions were used 242 times. Those most common were one-half, one-fifth, one-tenth, one-third, and one-fourth. In all except 43 cases, the denominators of fractions were less than ten. Twenty-four of the 43 ended in hundreds or thousands.
4. There were 434 decimal fractions; mixed decimals occurred 1,020 times. The most common form consisted of two integers and one decimal place. In general, almost any decimal up to four places was used.
5. Equations of the first degree and one unknown occurred seven times; equations with two unknowns occurred twice; fractional equations occurred six times; and algebraic addition occurred twice.

*This statement is meant to cover addition and subtraction of long expressions; multiplication and division of polynomials; manipulation of fractions with polynomial terms; squares and cube root of polynomials resulting in answers of more than two terms; involved factorization of polynomials; reduction of radicals (or fractional exponents) whose index is greater than three; operations with polynomials containing radicals; the factor and remainder theorems; and the binomial theorem, when the exponent is larger than five.

†Article by Williams in School and Society, October, 1921.

6. There were nine cases of substitution in formulae and 77 cases of ratio and proportion.
7. In general, it can be said that the arithmetic was complicated. In decimals 63 cases were found in which a decimal fraction or a mixed decimal was multiplied by an integer. In 16 cases mixed decimals were multiplied by mixed decimals, the most complicated being a five digit number with three decimal places multiplied by a five digit number with two decimal places. In 26 cases an integer was divided by a decimal and in 42 cases a mixed decimal was divided into another mixed decimal.

Charters summarized this study by saying: "The mathematical information needed in freshmen chemistry is limited to arithmetic and simple algebraic processes with a wide range of denominate number units."

Questionnaire Studies.

In this group seven studies will be reviewed, those by Jessup and Coffman, Wilson, Camerer, Moore (3 studies), and Woody.

Study 23. Jessup and Coffman—Opinion of Superintendents.*

The earliest of these significant studies was made by Jessup and Coffman. They wished to determine the attitude of city and county superintendents towards the elimination of certain doubtful subject-matter and the possible addition of new topics. They sent a letter to superintendents of schools in cities of 4,000 or over, and to every sixth county superintendent in the United States asking for opinion on various topics. They prepared two groups of topics. The first group of topics contained "cases of percentage," "compound proportion," "annual interest," and the like. The school officials were asked to underscore once the subjects which should receive slight attention, and underscore twice the subjects which should be eliminated. The second group of topics consisted of the fundamental operations as applied to "integers in fractions," "percentage," "stocks and bonds," and the like. In this case the superintendents were asked to underscore topics which should receive more attention than is usually given them.

Findings:

1. A little more than half the city superintendents responded and a little less than a quarter of the county superintendents.
2. Of those reported, a majority were in favor of increasing the emphasis on four fundamental operations on fractions, percentages, insurance, taxation, and thrift.
3. Between 10% and 50% of those reporting favored an increased emphasis on stocks and bonds, levies, banking, interest, building and

*Jessup, W. A., & Coffman, L. D., "The Supervision of Arithmetic," Ch. I & II. 1-38. See Third Yearbook, Dept. of Superintendence, P. 43.

loan associations, and profits. (It has not always been recognized, in quoting this finding, that this study borders on a "Yes" or "No" proposition, and that the number favoring the increase is insignificant.)

4. Over half of the superintendents voted to omit apothecaries' weight, root in square measure, compound proportion, and unreal fractions. (Was it clear to all superintendents what was meant by "root in square measure?" Wouldn't they have voted "Yes" on "finding square roots of numbers by a table of roots and powers?")

Study 24. Camerer—The Mathematics of Banking Used in Common Life.*

Camerer* tried to determine the minimum information that people in common life should have about banking. She made out an analysis of about thirty-five replies to a questionnaire which she had sent to bank employees in nine states, and also of replies to the same questions received from parents of pupils in the elementary school in the University of Iowa. A list of fifty-five items of banking information was submitted, with instructions to cross out those items of no importance, to check twice those that were very important, and to check once the less important items.

Findings:

1. The tabulation shows the items that were listed in the order of importance as determined by the number of double crossed checks. The order of the items for the first twenty is as follows:

(a) How to write a check.
(b) How and why to fill out a stub.
(c) When to cash a check.
(d) How to stop payment on a check.
(e) How to sign your name when indorsing a check.
(f) What to do in case a check is lost.
(g) How to indorse a note.
(h) How to write a negotiable note.
(i) How to indorse a check in full.
(j) How to use a bank book.
(k) How to make a deposit slip.
(l) How to find interest.
(m) Importance and purpose of savings banks.
(n) Importance and purpose of commercial deposit banks.
(o) How to use a promissory note.
(p) Responsibility of maker if note is lost.
(q) Certified checks.
(r) How to open an account.
(s) When notes are void.
(t) Legal rate of interest.

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*Camerer, Alice, "What Should be the Minimum Information about Banking?" Seventeenth Yearbook of the National Society for the Study of Education, Part I. Bloomington, Illinois, Public School Publishing Company, 1918. pp. 18-26.

2. The returns from parents included fifteen of the preceding items named above as the twenty most important items, but their ranking differed from that given the items by people experienced in banking.

Study 25. Wilson—The Opinion of Business Men.*

Wilson[1] tried to get the opinion of the business men of Connersville, Indiana, concerning the topics of arithmetic. He submitted two groups of topics to the business men and asked them to indicate the topics for which they had had considerable use in the preceding six months, and those for which they had had little or no use during the same time. He also asked them to indicate the topics that might well be omitted from the arithmetic work without material loss.

The second group of topics was prefaced by this expression: "Following are some topics which we did not study in the arithmetic as boys." The business men were asked to check any of the items in this list that appealed to them as worthy of a place in present-day arithmetic.

Findings:

1. As might have been expected, the business men reported little use for Wilson's first list of topics, and they suggested that these might well be eliminated from the course of study. The list consisted of:

Troy weight.	Foreign exchange.
Longitude and time.	Compound proportion.
The surveyor's table.	True discount.
The greatest common divisor.	Cases and 3 in percentage.
The least common multiple.	Compound interest.
Complex fractions.	Partial payments.
Cube root.	Partnership.

Compound fractions.

There was a scattering of support for cube root, compound proportion, cases in percentage, and surveyor's table.

2. As might also have been expected, all the business men agreed that every topic on Wilson second list should be included in the course of study. The list follows:

Saving and loaning money.
Mortgages.
Modern banking methods.
Building and loan associations.
Keeping simple accounts.
Investing money.
Bonds as investments.
Real estate as investments (cheap rentals, good residence property, business blocks, farm lands as investment).
Mark of a good investment. (It is estimated that the get-rich-quick concerns fleece the American people out of $60,000,000 a year.)
Taxes, levies, public expenditures.
Profits in different lines of business.

*Wilson, Guy M. "Connersville Course in Mathematics" (First Edition 1910-1911). Baltimore, Md., Warwick and York, pp. 6-10.

Study 26, 27 and 28. Moore—The Arithmetic of Every Day Life.*

Study 26.

Moore reports three studies. In his first study he tried to determine how much arithmetic is used in everyday life by sending a questionnaire to men in various occupations. He asked them such questions as "Do you personally have occasion to compute simple interest?" He received 799 replies to his questionnaire.

Findings:

From these replies it is concluded that the minimum essentials should include the following:

(1) Addition—five addends, five figures in width.
(2) Multiplication—the multiplicand, five figures in width; the multiplier, four figures in width.
(3) Division—the dividend, five figures in width; the divisor, four figures in width.
(4) Fractions—halves, thirds, fourths, fifths, eighths, with tenths.
(5) Decimals of three places.
(6) Simple interest.
(7) Percentage.

Study 27.

The second study reported by Moore was an attempt to supplement the first by sending a list of topics in arithmetic, such as simple cash accounts, drawing to scale, and the like, to the individuals included in his first study. He requested that they give to the children in the home a statement of what use they had made of arithmetic each day for ten consecutive days, and have the children check the topics employed.

Findings:

To this questionnaire Moore received 314 replies and from his analysis he concludes that the list of minimum essentials should include the following:

(1) Simple cash accounts, or family expense accounts.
(2) Cash, checks, or bills.
(3) Addition of fractions.
(4) Multiplication of fractions.
(5) Subtraction of fractions.
(6) Banking.
(7) Division of fractions.

Study 28.

In his third study Moore tried to determine the arithmetic needed by employees in fifty business firms in Los Angeles by asking four questions.

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*1923 Moore, Ernest C., "Minimum Course of Study." Chapter I, Arithmetic. Ph. 1-41. See also Woody in Third Yearbook of the Department of Superintendence, page 56.

Findings:

He secured 24 replies and from these it appears that (1) the arithmetic needed on entering a firm should include the fundamentals (as applied to whole numbers, common fractions, decimals and percentage); (2) the chief weaknesses manifested were accuracy in the material listed in (1) above; (3) the suggestions made for correcting the mistakes, in addition to greater accuracy, were short cuts, mental arithmetic, and "teach the why"; (4) in response to what should be eliminated the replies were "everything except the four fundamentals, decimals, interest, and discount."

Study 29. Woody—The Mathematics of Business Transtions.*

Woody tried to determine to what extent calculating and measuring devices were used in business houses, and to learn something about the nature of the arithmetic that is used in these business transactions. He interviewed forty business men in Ann Arbor concerning mechanical devices, short cut methods, and the nature of arithmetic used by salesmen and consumers. A uniform list of questions was used as the basis for his interviews. His distribution of business houses seems to be about as wide as it could be in a town of approximately twenty thousand people.

Findings:

Woody summarizes his findings as follows:

"(1) Each of the business houses made much use of mechanical calculating devices. Each firm had at least one cash register, and some had as many as ten. Almost all firms had at least one adding machine, and the banks had from six to eight machines. The banks used bookkeeping machines. Computing scales were used in all grocery stores, all meat markets, all drug stores, and in all candyshops. In one grocery store two cheese cutters were operated on the computing plan.

"(2) Many tabular devices were utilized in order to reduce the amount of tabulation. The banks used bond and interest tables. The news company used a 'price table' to tell the price of the different magazines or combinations of magazines. The drug stores used 'percentage solution' tables whereby it was possible to find the different amount of a chemical to make a solution of a particular strength. The hardware had 'screen tables' for finding the areas of screens of different lengths and widths, and discount tables for determining the cost price from the list price in wholesale catalogues. The book stores used a

*Woody, Clifford, "The Use of Calculating and Measuring Devices in Business and Their Influence on the Use of Arithmetic." Unpublished study made in cooperation with Leander Beach. On file in the office of the Bureau of Educational Reference and Research, University of Michigan.

'printer's table' for determining the number of sheets per pound according to quality. In fact, almost every firm used tabular devices of some kind.

"(3) Several 'short-cut' methods were discovered. In computing interest at six per cent when the interest book is not used, point off two places in the principal and multiply by half the number of months. In figuring the selling price on an article when there is a discount, subtract from 100 the discount, point off two places in the remainder and multiply by the original price. A system of allegations was used by the druggists.

"(4) In any case the goods were marked to avoid complicated calculations. Firms are now avoiding marking in fractions and employ the unit or decimal system to a great extent. They also avoid the sliding scales and sell at so much per unit or multiple of this unit, or group of units. An illustration of the unit or group of units plan is: Selling six bars of soap for twenty-five cents, but charging thirty-five cents for eight bars.

"(5) Many of the managers insisted that neither their employees nor the purchasers did much actual calculation. In certain stores like restaurants and ice cream stores, where articles were sold in combination, the clerks learned the sale price for the combination of articles, e. g., they learned the regular price for a sandwich, coffee and a piece of pie was forty-five cents and did not have to add the price of the different items to get the total. The managers estimated that only about twenty-nine per cent of the buyers ever add the cost of different articles purchased to find the total amount purchased, and that only about 66.6 per cent of their customers ever check the change given them.

"(6) These business men insisted that the school needs to emphasize the four fundamentals. They felt there is great need for 'mental' arithmetic. Some emphasized the need for training involving interest, discount, and the metric system. The particular needs emphasized by the business men depended largely on their type of business.

"(7) The business men agreed that the greatest number of mistakes made in the buyer's calculations involve making change. The buyer does not understand the 'make-change' method and gets lost.

"(8) The outstanding conclusion drawn from this investigation is that little calculation in arithmetic is actually done."

SUMMARY TABLE—THE FIRST CRITERION

We now propose to summarize the evidence from these 29 objective investigations which attempt to show the social use of mathematics. What curriculum in mathematics would we get if we accepted only those elements for which some kind of a positive case can be made? In the following table these items are listed, together with names in parentheses which indicate the references where possibly the strongest evidence may be found:

1. High skill in the ability to estimate answers for checking results secured by automatic mechanical devices in the business world. (Woody)

2. Reading large numbers, probably to billions. (Adams)
3. Knowledge of common units of foreign money—the English pound, the French franc, and the like. (Adams)
4. Familiarity with a large variety of denominate numbers. (Adams, Schorling, Williams, Woody)
5. Familiarity with a long list of units of measure, probably more extensive even than found in our school texts. (Adams, Schorling)
6. The rational use of significant figures. (Nat. Com.*)
7. Numerical computation with approximate data. (Nat. Com.)
8. The meaning and use of the elementary concepts of statistics. (Nat. Com.)
9. Computation with short-cut methods. (Nat. Com.)
10. Taking square root of arithmetic numbers. (Torndike)
11. Understanding of the Use of Mechanical Devices—weighing, measuring, computing, and the like. (Woody)
12. Skill in the extensive use of tables—bonds, interest, percentage solution, screen, discount, printers, and the like. (Woody)
13. Familiarity with the metric system. (Nat. Com., Schorling, Woody)
14. Graphical representation of statistical data. (Nat. Com.)
    (a) Bar, line, and circle graphs to represent ratio and dependence. (Nat. Com.)
    (b) Reading graphs showing relation of one variable to another. (Thorndike)
    (c) Critical faculty in reading and in evaluating graphs. (Thorndike)
    (d) Presentation of laws by mathematical graphs. (Nat. Com., Thorndike)
15. The reading of geometric figures by means of letters designating points. (Gallaway)
16. Familiarity with numerous geometric terms, a list probably far more extensive than found in the mensuration of arithmetic. (Gallaway, Schorling)
17. Understanding of an extensive list of geometric concepts, most of which can probably be taught to children by simple intuitive and experimental methods. (Schorling)
18. The use of simple geometric constructions:
    (a) Drawing a line segment of a given length. (Gallaway)
    (b) Drawing a line parallel to a given line.
    (c) Drawing a line perpendicular to a given line.
    (d) Drawing a circle or an arc of a circle having given the center and the radius (Gallaway)
19. Command of the fundamentals when applied to integers:
    1. For addition the following suggestions seem important:
       (a) Addends of four digits. (Charters)
       (b) Six addends and not more than five places. (Woody)
       (c) Five addends with five digits. (Moore)

*Hereafter Nat. Com. will refer to the National Committee on Mathematical Requirements.

2. For subtraction not more than four places in the minuend. (Woody)
3. For multiplication we find:
   (a) The multiplicand five digits, and the multiplier four digits. (Moore)
   (b) The multiplicand twelve or less with multiplier three places or less. (Charters, Woody)
4. Division:
   (a) The dividend five digits and the divisor four digits. (Moore)
   (b) Three place numbers in dividend are most common. (Woody)

20. High skill in applying the fundamentals to common fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 16, 323. This list, brief as it is, allows for a "safety factor"—it is considerably longer than the one suggested by Thorndike and much more extensive than that of the National Committee on Mathematical requirements (Schorling, Wilson, Wis.).
21. To guide us in applying fundamentals to mixed numbers the following are significant:
   (a) Mixed decimal by mixed decimal (16 cases). (Williams)
   (b) Multiplying two fractions. Gallaway)
   (c) Multiplication by a fractional, integral, or mixed number of dozens. (Mitchell)
   (d) Integer by decimal. (Williams)
   (e) Mixed decimal divided by mixed decimal. (Williams)
   (a) Addition of any number of twelfths to integers and mixed numbers. (Mitchel)
   (g) Reducing mixed numbers to improper fractions. (Gallaway)
   (h) Almost any decimal up to four places. (Williams)
   (i) Goods frequently marked in decimal system. (Woody)
   (j) Multiplying five digit number with three decimal places by five digit number with two decimal places. (Williams)
22. Ability to use simple business forms: family expense accounts, checks, bills, notes, deposit slip. (Camerer, Moore)
23. Knowledge of the use of discount, interest, opening bank account, etc. (Camerer, Moore)
24. Knowledge of saving and loaning money, mortgages, building and loan, simple accounts, thrift, bonds and real estate investments, taxes, and levies. (Wilson)
25. Acquaintance with insurance, taxation, and thrift. (Jessup and Coffman)
26. Ability to interpret percentage in a great variety of ways. (Adams)
27. Ability to find per cent of a given number. (Williams)
28. Ability to interpret ratio in a great variety of ways. (Adams, Nat. Com.)
29. Ability to use proportion. (Nat. Com., Williams)
30. Ability to use the simple machinery of algebra:
   (a) Use axioms. (Nat. Com., Thordike)

(b) Read symbols.
(c) Know the meaning of the omission of time sign.
(d) Know the meaning of the omission of one as a coefficient.
(e) "Letting — = the number of —." (Thorndike)

31. Ability to translate a verbal statement into symbolic. (Gallaway)
32. Ability to deal with a single parenthesis. Thorndike)
33. Ability to deal with a simple fractional exponent and transformation of the type $\sqrt[3]{a} = a^{1/3}$. (Thorndike)
34. Knowledge of positive integral exponents. (Thorndike)
35. The ability to interpret negative numbers. (Nat. Com., Thorndike)
36. The evaluation of simple algebraic expressions. (Nat. Com., Rugg and Clark)
37. The ability to apply fundamentals to simple algebraic terms, as for example, multiplication of two binomials. (Rugg and Clark)
38. The ability to apply fundamentals to simple fractions. (Rugg and Clark)
39. The ability to clear a proportion of fraction—great skill in the use of the proportion form of equation. (Thorndike)*
40. The ability to form correct proportions. (Rugg and Clark, Thorndike)
41. The ability to do the simplest cases of factoring. (Rugg and Clark, Thorndike)
42. Understanding of the linear function, $Y = mx + b$. (Nat. Com.)
43. Ability to use the equations of the first degree and one unknown. (Rugg and Clark)
44. Ability to use simultaneous linear equations and two unknowns. (Nat. Com., Williams)
45. Ability to use fractional equations. (Williams)
46. The ability to use quadratic equations in one unknown. (Nat. Com.)
47. The ability to obtain L. C. M. by inspection in the case of a fractional equation with simple numerical denominator. (Rugg and Clark)
48. Ability in the use of the formula:
    (a) Ability to read simple formulas.
    (b) Ability to solve simple formulas.
    (c) Ability to evaluate simple formulas.
    (d) Ability to construct simple formulas.
    (e) Ability to change the subject of a formula.
    (f) Ability to substitute in a formula. (Nat. Com., Thorndike)
49. Some skill in the solution of verbal problems. Those dealing with:
    (a) Proportion, such as mixtures. (Thorndike)
    (b) Buying and selling goods. (Wilson)
    (c) Labor and wages. (Wilson)
    (d) Interest. (Wilson)
    (e) Rent.* (Wilson)
    (f) Insurance. (Wilson)
    (g) Measurement of capacity.* (Wilson)

*Contradicts other evidence but the writer is convinced that Thorndike is correct.

(h) Estimates of building materials.* (Wilson)
50. Ability to use logarithms in computation.
51. Ability to use sine, cosine, and tangent in the solution of simple problems of the right triangle. (Nat. Com., Rugg and Clark)

The Inadequacy of Any Single Objective Study.

It ought to be unnecessary to point out that any one of the so-called scientific studies now available cannot serve as the basis for the selection of subject matter. But it is not infrequent that much publicity is given to a single study holding out the hope that our curriculum problems are being solved by it alone. In the summer of 1924 the newspapers of our Nation's capitol, at the time of a great teachers' convention, gave much publicity to one of the studies reviewed in this chapter in a way that was probably very deceptive to the public.

Let us examine two studies to illustrate this inadequacy. Consider first the most recent study by Adams in which he tried to discover the mathematical uses in general reading. He finds very little trace of algebra, geometry, or trigonome- He finds very little trace of Algebra, geometry, or trigonometry. Bobbitt, who sees Adams pull in his net, concludes that these are the only fish in the sea. A study† presented in this chapter shows that if you shift your position only slightly and cast the net again, you catch an entirely different kind of fish and many more of them.

Next, let us examine what is probably the best known of the studies, namely Wilson's collection of problems. Wilson assumes that he has a random sampling. He found not a single trace of stocks or bonds. At the time that he made this study, according to government reports there were over 4,400,000 stockholders of corporations. It has not been possible to find an estimate of the number of bond holders, but it must have been very great, for municipal bonds, township pike bonds, and the like, had been in common use long before Wilson's study was made. There is here then, clear evidence that he did not have a complete picture of mathematical needs, but for the sake of argument let us make the false assumption that people did not in common life possess stocks and bonds at the time he collected his problems. Before Wilson analyzed his problems and printed his conclusions the Govern- analyzed his problems and printed his conclusions, the Government was making a nation-wide drive to get all people to in-

†Little Difference in Type between Problems Collected from the City and those Collected from the Country—Wise.
*See Study Number 3.

vest in Liberty Bonds. Mathematics must not only meet present needs, but it must teach those fundamental principles that will make it possible to anticipate needs.

A third factor is one suggested by Thorndike*. The question, "What uses do people make of mathematics?" is not the only issue. It is perhaps even more important to inquire: "What use could they make with great economy and profit?"

From these illustrations it is quite clear no single study can serve as a sole basis for choice of materials. Hence, we have attempted to build a summary table in this chapter which uses the individul studies as side lights on the general problem.

It must be admitted that the evidence for some parts of algebra is far from convincing. In chapter V where this summary table is used as one of the five criteria, we shall reject all objective evidence relating to algebra other than that resulting from Thorndike's investigations and the outcome of the writer's manipulation of the table on page 00.

The Second Crierion

Practice as Determined by an Inventory of Courses of Study.

A very valuable contribution to our problem has become available in Walter Scribner Guiler's Doctor's dissertation.† Guiler made a detailed analysis of thirty courses of study in arithmetic covering the first eight years work. He accepted any element in arithmetic which appeared in three or more of the thirty courses. The courses chosen were those frequently mentioned by leading educators in response to a questionaire as possessing greatest merit.

Although Guiler has made a valuable contribution by his careful work, he seems to have attached too much importance to the outcome. He accepts his course of study thus obtained as "best practice." He gets practice, not "best practice." The probability is high that if Guiler were to repeat his study he would get a very different course of study.

If each course of study represented independent thought, the objection might not be so valid. The fact is that a course of study often duplicates largely a committee report or some other course of study, or possibly it is made to fit a textbook. The three most detailed and helpful courses of study in arithmetic recently published have been built to fit specific textbooks.

*See "Psychology of Algebra," the chapter on Uses of Algebra.
†Just finished and filed at the University of Chicago.

Progress would indeed be slow if "best practice" could not be modified until a new element had been accepted by three prominent cities. In fact it seems never to have occurred to Guiler that logically there wouldn't be any progress under his technique. All schools would automatically adopt the practice found in three or more courses of study and thus dispose of all troublesome questions for all time.

Guiler's study furnishes us the practice for grades seven and eight. Using his technique for the most part, we shall presently proceed to extend his work through the ninth grade. The outlines for these three courses will constitute our third criterion in the selection of subject matter.

It is believed that such an outline is a valid criterion, for in the long run the more worth while materials undoubtedly tend to persist. When courses of study are being made in a democratic way by committees of teachers, it would seem that the less desirable materials would be rejected.

The outlines which follow are quoted from Guiler's thesis. The numbers in parentheses indicate the number of courses which listed each objective. The grade placement of the objectives was determined by their modal placement in the original course of study.

OBJECTIVES FOR GRADE VII

Decimals

1. Ability to add demicals used in business (3).
2. Ability to subtract decimals used in business (3).
3. Ability to multiply decimals used in business (3).
4. Ability to divide decimals used in business (3).

Denominate Numbers

1. Ability to read per cents expressed as integers (3). common use (6).
2. Ability to subtract any units of denominate numbers in common use (6).
3. Ability to multiply any units of denominate numbers in common use (6).
4. Ability to divide any units of denominate numbers in common use (6).

Percentage

1. Ability to read per cents expresed as integers (3).
2. Ability to find per cent equivalents of fractions used in business (3).
3. Ability to find a number when a given per cent of it is known (4).

Applications of Percentage

1. Ability to compute simple interest (20).
2. Ability to compute rate in simple interest (4).
3. Ability to compute compound interest (6).
4. Ability to compute bank discount (11).
5. Ability to compute simple partial payments (4).
6. Ability to use interest tables (8).
7. Ability to understand and appreciate the value of interest as a social institution (3).
8. Ability to find commercial discount (11).
9. Ability to find rate of commercial discount (4).
10. Ability to understand commercial discount as a social institution (3).
11. Ability to find profit or loss (11).
12. Ability to find per cent of profit or loss (5).
13. Ability to find commission (16).
14. Ability to find the rate of commission (7).
15. Ability to solve problems involving finding rate of commission (3).
16. Ability to understand how rate of insurance is affected by different factors (3).
17. Ability to solve problems involving finding of premium (4).

Money

1. Ability to open a checking account (19).
2. Ability to make bank deposits (21).
3. Ability to write a check (23).
4. Ability to balance a check book (19).
5. Ability to understand and appreciate the value and operation of a bank as a social institution (7).
6. Ability to plan a personal budget (7).

Mensuration

1. Ability to recognize a quadrilateral (5).
2. Ability to find the volume of a sphere (4).)
3. Ability to find the volume of a triangular prism (8).

Generalized Abilities

1. Ability to express oneself accurately and freely in arithmetical language (6).

Equations

1. Ability to use the equation in numerical work (14.)
2. Ability to find the unknown quantity (3).

Nomenclature and Symbols

1. Ability to use the nomenclature of interest (4).

OBJECTIVES FOR GRADE VIII

Denominate Numbers

1. Ability to sense the relation between longitude and time (4).
2. Ability to transmute the common units of the metric system into U. S. equivalents (5).

Applications of Percentage

1. Ability to find the cost of stocks and bonds at a given market value (3).
2. Ability to understand and appreciate the value of stocks and bonds as social institutions (6).
3. Ability to understand and appreciate the value of insurance as a social institution (14).
4. Ability to understand and appreciate the value of taxation as a social institution (12).

Money

1. Ability to do personal accounting (22).
2. Ability to utilize various agencies, such as savings account, mortgage, etc., in saving and investing money (26).
3. Ability to utilize various agencies, such as bank, insurance policy, collateral, etc., in borrowing money (11).
4. Ability to utilize various agencies, such as draft, express money order, postal money order, etc., in sending money (18).

Mensuration

1. Ability to find the perimeter of: (a) triangle (10), (b) circle (7).
2. Ability to find the area of:

triangle (24)	parallelogram (10)	prism (7)
rectangle (21)	cylinder (9)	cube (3)
circle (19)	square (8)	

3. Ability to find the volume of:

rectangular prism (23)	pyramid (8)
cylinder (15)	cone (6)

4. Ability to solve problems involving finding the perimeter of:
   (a) circle (3), (b) parallelogram (3)
5. Ability to solve problems involving finding the area of:
   (a) triangle (4), (b) circle (3), (c) parallelogram (3)

Practical Measurements

1. Ability to measure and estimate surface for purposes of making computations for:
   (a) plastering (10), (b) painting (10),
   (c) carpeting (10)
2. Ability to measure lumber (11).
3. Ability to find the capacity of:
   (a) rectangular forms (4), (b) cylindrical forms (4)

Generalized Abilities

1. Ability to use an economical method of procedure (5).

Ratio and Proportion

1. Ability to express numerical relations as a ratio (4).
2. Ability to express numerical relations as a proportion (4).

Nomenclature and Symbols

1. Ability to use the common nomenclature of insurance (8).

NINTH GRADE ALGEBRA

The following ninth grade algebra outline is the outcome of an inventory of ten state courses of study.* The result of the tabulation is, as might have been expected, the formal, traditional course. There are only a few traces of the re-organization movement.

As in the case of Guiler's outline, an element in algebra which appears in three or more of the state courses of study is accepted. The figures at the left of the items indicate the frequency of the element, whereas the row of figures at the right of each element shows the particular courses in which the item was found. Thus a "7" at the left shows that seven of the ten courses of study made particular mention of the item in question, whereas the row of figures "4-8-10" indicates that the element concerned was found in the courses of study, for Texas, 1922; for Michigan, 1923; and for Indiana, 1923.

*(1) Illinois, 1918; (2) Pennsylvania, 1914; (3) Utah, 1918; (4) Texas 1922; (5) Florida, 1924; (6) Missouri, 1919; (7) Virginia, 1924; (8) Michigan, 1923; (9) New Mexico, 1916; (10) Indiana, 1923.

THE ALGEBRA LIST

(5) I. Correlation and comparison of algebraic and arithmetic processes. 1-3-5-6-10.

(4) II. Emphasis upon the language of algebra. 2-4-7-10.

III. Fundamental operations.

(10) 1. To algebraic expressions not involving negative numbers.

(10) 2. To positive and negative numbers.

(5) 3. Parenthesis removal and inclosure. 1-3-5-7-9. (No limits suggested by as many as three courses.)

IV. Special products and factors.

(10) 1. $c\,(a+b)$ all courses.
(10) 2. $(a+b)\,(a-b)$ " "
(10) 3. $(x+a)\,(x+b)$ " "
(10) 4. $(ax+b)\,(cx+d)$ " "
(10) 5. $(a\pm b)^2$ " "
(4) 6. $(x\pm y)\,(x^2\mp xy+y^2)$ 4-5-6-9

V. Equations.

(3) 1. Practice in translation from verbal to algebraic expression. 1-4-10.

(8) 2. Emphasis in solution of linear equations by fundamental processes rather than by transposing. 1-2-5-6-7-8-9-10.

(8) 3. Solving a pair of linear equations by addition and subtraction. 1-2-5-6-7-8-9-10.

(6) 4. Solving by substitution. 1-2-5-7-9-10.

(6) 5. Solving by graphs. 1-2-5-7-9-10.

(4) 6. In three unknowns. 1-5-7-9.

(3) 7. Quadratics in one unknown solved by factoring. 1-6-9.

(3) 8. Quadratics in one unknown solved by completing the square. 1-6-10.

(3) 9. Solving literal equations. 3-7-9.

(3) 10. Solving fractional equations leading to quadratics. 7-9-10.

(4) 11. Checking quadratic equations by substitution. 1-4-8-10.

VI. **Fractions.**

(10) 1. **Fundamental operations. All courses.**

(3) 2. Reduction of easy complex fractions. 4-6-8.

VII. Exponents.

(10) 1. Positive integral. All courses.

(3) 2. Fractional. 2-6-9.

(3) 3. Negative. 2-7-9.

VIII. Formulas.

(6) 1. Meaning and use. 3-5-7-8-9-10.

(3) 2. Graph of. 7-8-10.

(3) 3. Evaluation. 7-9-10.

(4) 4. Function concept illustrated by formulas. 1-7-8-10.

IX. L. C. M. and H. C. F.

(4) 1. By inspection. 1-2-9-10.

X. Radicals.

(7) 1. Fundamental operations applied to simple radicals. 1-2-5-6-7-8-9.

(3) 2. Square root of trinomials. 1-5-6.

(4) 3. Square root of polynomials. 2-7-9-10.

(6) 4. Square root of arithmetical numbers. 1-5-6-7-8-10.

The lists for the seventh and eighth grades from Guiler and the preceding list for ninth grade algebra are accepted as reflecting *practice,* and constitute the second criterion to be used in Chapter V.

CHAPTER IV

AN INVENTORY OF THE CONTENT OF SEVEN SERIES OF JUNIOR HIGH SCHOOL MATHEMATICS TEXTS (THE THIRD CRITERION)

Is an Inventory Worth While?

It might be inferred that it would also be futile to make a careful inventory of the various junior high school texts, but this does not necessarily follow. The early junior high school books represent an effort to apply a new philosophy of secondary education to a special field. For some authors this factor may have been operating without their recognition. This chapter will inventory the material in the first seven sets of junior high school books to appear. These books represent the work of eleven authors, and, without exception, they possessed unusual insight. Theirs was the pioneer task. In all probability, the same effort to produce more conventional materials would have yielded greater financial returns. Each series offers a lay-out of materials that is distinctly different from conventional practice in these three grades. Even a casual and superficial survey of any one of these seven series of books would seem to give conviction that here is a genuine effort to get something better, something more worth while and more interesting for boys and girls in these grades. For these reasons, the content of these seven books, the first in the field, is accepted as one of five criteria in the selection of materials for grades seven, eight, and nine.

The Offerings Are Distinctly Different.

Let us first take a general view of the offerings. The following table shows the organization by semesters or half years. Text A, for example, offers arithmetic in the first half of the seventh year and intuitive geometry in the last half; it has the pupil study algebra the first half of the eighth and arithmetic the last half; algebra with a unit of trigonometry is given in the first half of the ninth, and the last half is taken up by plane geometry.

TEXT	SEVENTH		EIGHTH		NINTH	
	1st half year	2nd half year	1st half year	2nd half year	1st half year	2nd half year
A.	Arith.	Intuitive Geometry	Algebra	Arith.	Algebra and Trig.	Plane Geometry
B.	Arith.	Geometry	Arith.	General Math.	Algebra	Algebra
C.	General Math.	General Math.	General Math.	Arith.	Algebra	Algebra and Trig.
D.	Arith.	Arith.	General Math.	General Math.	Algebra	Plane Geometry
E.	General Math.	General Math.	Algebra	Algebra	Arith.	Arith.
F.	Arith.	Arith.	Int. Geom.	General Math.	General Math.	Gen. Math. and Trig.
G.	General Math.	General Math.	General Math.	General Math.	Algebra	Plane Geometry

This table reveals an astonishing degree of variation. It would seem difficult for a parent to predict what large field of mathematics his child will be expected to study. Election in mathematics might mean arithmetic, geometry, algebra, or general mathematics. This is even true of the last half of the ninth grade. Three of the books offer plane geometry, one, arithmetic only, one, general mathematics including a unit of trigonometry, and two offer algebra, each including a unit of trigonometry. Is there any wonder that school executives, supervisors, committees working on courses of study, to say nothing of the beginning teachers, are confused by the wholly different offerings presented? We have here evidence of distinctly different hypotheses concerning the organization of materials for these grades.

A More Detailed Comparison.

It will be helpful to search for the material that is common to a considerable number of texts. In particular, for what concepts are illustrations and explanations furnished, and what elements are considered sufficiently important to be given recall? The following articles will attempt to find the material for which these two things are true.

What Formulas Do the Texts Attempt to Teach?

Teachers of secondary school mathematics quite generally agree that one of the important and enriching developments in recent years has been the emphasis which has come to be placed upon the formula. In this particular part of our study,

we are not concerned with the construction of a formula and its transformation from one form to another. The question before us is "what formulas is the pupil *expected to know?*" At first glance this search would seem hopeless. Taken as a single group, the twenty-one texts present hundreds of different formulas. The second book of series A alone contains not less than fifty different formulas for the eighth grade. These are given without explanation nor are they practiced later in very many cases. Practically every series presents numerous formulas of great difficulty. Here are ten chosen from one eighth grade book:

$$\frac{W}{P}=2R(R-S); \qquad R=\frac{S+h^2}{2h}; \qquad L=\frac{13}{8}(D+d)\,2c;$$

$$g=\frac{1}{924}\,d^2h; \qquad c=.004d^2+1.4w; \qquad T=D-\frac{1.3}{N};$$

$$r=\frac{P}{c+P+o}; \qquad A=\frac{\pi}{3}\,Kd; \qquad F=\frac{wp}{2\pi r} \text{ and}$$

$$V=\frac{5^{\frac{2}{3}}}{6\pi\,\frac{1}{2}}$$

We repeat that it would be possible to quote scores of formulas no less difficult from these texts written for eighth grade pupils. In the light of the data presented in Chapter II, dealing with the tasks that children can do at this level, the difficulty of these formulas seems preposterous. Surely the college professor whose experience with children lies far in the past falls into grievous errors. The educational public has justly become impatient with the usual method of textbook making. All too frequently textbooks have been written quite removed from the activities of pupils for whom they are intended, published without experimental testing, and sold for a number of years, until an avalanche of criticism compels a revision. Such practice constitutes an unmeasured, uncontrolled experiment though it commonly is not recognized as such.

What list of formulas do we get if we lay down these two criteria? (1) A formula must appear in four of the seven series concerned. (2) There must be at least some slight hint or clue given to the pupil as to what the formula is about, together with practice and later recall. By applying these two criteria to the twenty-one books concerned we get the following table:

FORMULA FOUND IN FOUR DIFFERENT SERIES OF TEXTS

		A	B	C	D	E	F	G
(1)	7, 18 $A=\pi r^2$ (circle)	1, 2, 3	1, 2, 3	1, 2,	2	1, 2, 3	1, 2, 3	1, 2, 3
(2)	7, 17 A =bh (rectangle)	1, 2, 3	1, 2, 3	1	2, 3	1, 2, 3	1, 3	1, 2, 3
(3)	7, 17 A =½bh (triangle)	1, 2, 3	1, 2, 3	1, 2	2	1, 2, 3	2, 3	1, 2, 3
(4)	7, 17 A =½h (b+B) (trapezoid)	1, 2, 3	1, 2, 3	1	2	1, 2, 3	1, 2, 3	1, 2, 3
(5)	7, 17 $C=2\pi r$ (circle)	1, 2, 3	1, 2, 3	1, 2	2	1, 2, 3	1, 3	1, 2, 3
(6)	7, 17 V =hwl (prism)	1, 2, 3	1, 2, 3	1	2, 3	2, 3	3	1, 2, 3
(7)	6, 12 $V=r^2h$ (cylinder)	1, 2, 3	2, 3	1	2, 3	2, 3	2, 3	
(8)	6, 11 A =bh (parallelogram)	1, 2	1, 2, 3	1		1	1, 2	1, 3
(9)	6, 8 $S=2\pi rh$	1		1	2	2, 3	2	2
(10)	6, 9 V =bh (prism)	2	1, 2, 3	1	2	2, 3	2	
(11)	6, 7 I =p rt (interest)		3		1	2	2, 3	3
(12)	5, 10 $C=\pi d$ (circle)	1, 2, 3	1, 2, 3		2		3	3
(13)	5, 7 $V=1/3bh$ (pyramid)	2	2, 3	1	2		2, 3	
(14)	4, 8 $A=X^2$(square)		3		2	1, 2, 3	2, 3	3
(15)	4, 7 V =bh (cylinder)	1, 2	1, 2, 3	1	2			
(16)	4, 7 $V=1/3\pi r^2h$ (cone)	2, 3		1, 2	2		2, 3	
(17)	4, 7 $S=4\pi r^2$ (sphere)	3	2, 3	1, 2			2, 3	
(18)	4, 6 $S=\pi rs$ (cone)			1, 2	2	3	2	
(19)	4, 6 p =rb (percentage)					2, 3	1, 2	3
(20)	4, 6 d =rt (distance)			3	3	3	3	3
(21)	4, 6 $V=4/3\pi r^3$ (sphere)		2, 3	1, 2			3	3

Note that there are only twenty-one items that meet our conditions.

Note that the items are arranged in order of importance. This is suggested by the figure in the left hand column. For the first item the "7" means that this item is developed somewhere in each of the seven series. The next figure, "18," means that it appears somewhere in eighteen of the twenty-one books in the series. It is obvious that there are only six formulas sufficiently important to be dealt with so as to meet our conditions in each one of the seven series. The figures in the right hand column indicate the importance that the items held in the author's mind. Thus, a "1, 2, 3," would indicate that the author treated the item in each of the three books. In like manner, if only a "2" appears it means that the formula is used only in the book for the eighth grade, whereas if "1" is used, it indicates that the item appears only in the seventh grade book. A blank space in the right hand column shows that the corresponding series of books does not develop the formula in question. The words in parenthesis indicate to what the formula refers.

Of these twenty-one items, all but three, namely, the "interest law," the "percentage rule" and the "distance formula," express geometric relations. This suggests that, when an author tries to explain a formula so that he may dare hope to have his pupils get some control, he turns to the geometric relations as the most concrete material. Other things being equal, if a graphic illustration accompanies a situation involving number relations more people will understand and appreciate it.

Social Economic Material.

Consider next the various business forms. As in the study of formulas, the number of different forms presented is very great—many appearing in only one of the twenty-one books. The following is a list from a *single* book on only *two topics*: partial payments, installment payments, customer's ledger, time deposits, daily balance for large firms, clearing house forms, federal reserve banks, bank examiners, bank statements, express money order, postal money order, certified check, foreign exchange, money by telegraph, commercial payment, traveler's check, letter of credit, sight draft, bonding company, express rate, freight rate, passenger rate, bill of lading, inter-road charges, telephone rates, telegraph rates, parcel post rates, mail (1st, 2nd, 3rd and 4th class) rates, time clocks, pay sheets, sales accounts, purchase accounts, buying expenses, selling expenses, corporation profits, corporation losses, coupon notes.

Space prevents the publication of the full list of items found in the different books but the diversity shows the very great need for greater agreement as to what we wish pupils to know.

Before applying the criteria in the section dealt with formerly, we need to amend these so as to omit "practice and re-call" or we would have very few items in our next table. Apparently authors are well aware that pupils are not as a rule deeply interested in this material for which they usually lack maturity and experience. These items are usually developed briefly, given a page or two of practice and *dropped.* It is as if we hear a sigh of relief from the author with a "here, that's done; at least they can't accuse me of leaving this topic out." Accepting again mention in four texts with at least some explanation and illustration as our criterion, we get the table which follows. These items are again roughly arranged in the order of importance; the figures in the different columns attempt to express the same as those in the previous table. Note that Text C has much of this material in the third book of the series, that Text E has it in book two and Text A presents in in the first book. Surely the problem of grade placement has not been solved.

SOCIAL-ECONOMIC MATERIAL INCLUDED IN FOUR OR MORE SERIES OF TEXT BOOKS

	A	B	C	D	E	F	G
7, 11 Bills	1, 2	1, 2	1, 3	1	2	1	1, 2
7, 10 Commission and brokerage	2	1, 2	3	1	1, 2	1, 2	1
7, 9 Check	1, 2	1, 2	3	1	2	2	2
7, 9 Indorsement of check	1, 2	1, 2	3	1	2	2	2
6, 10 Discount (commercial)	1, 2	1, 2	1	1	1, 2		1, 2
6, 9 Promissory notes	1	1, 2	3		2	1, 2	1, 2
6, 8 Deposit slips	1, 2	1, 2	3	1		2	2
6, 8 Balancing accounts	1, 2	1, 2	3	1	2	1	
6, 9 Personal accounts	1, 2	1, 3	3	1	2	1	
6, 7 Indirect taxes	2	2	1, 3		2	2	2
6, 6 Direct taxes	2	2	3		2	2	2
6, 6 Stock (common and preferred)	2	2	3		2	2	2
6, 6 Insurance (property, life)	2	2	3		2	2	2
6, 7 Compound interest	1	1, 2	3		2	2	2
6, 7 Savings banks	1	1, 2	3		2	2	2
5, 7 Bank discount	1, 2	1, 2	3			2	2
5, 7 Household accounts	1, 2	1, 2	3	1	2		
5, 8 Receipted bills	1, 2	1, 2		1	2		1, 2
5, 7 Bills with discounts	1, 2	1, 2	3			1	2
5, 5 Bank accounts	1	1	3	1			2
5, 5 Stub of check	1		3	1	2		2
5, 5 Time note	2	2	3			2	2
5, 5 Bonds (coupon, registered)		2	3		2	2	2
4, 6 Invoice	1, 2	1	3			1, 2	
4, 5 Bank draft		1, 2	3			2	2
4, 5 Demand notes		2	3			2	1, 2
4, 5 Household budgets	1, 2			1	2	1	
4, 4 Inventory	2	2	3	1			
4, 4 Postal saving systems	1	1	3			2	
4, 4 Business accounts	2	2		1			2
4, 4 Discounting notes (by bank)	2	2				2	2
4, 5 Indorsement of promissory note	1		3			2	1, 2

In this table we have twenty-eight items. Of these twenty-eight items only four, dealing in fact only with three topics, occur in all seven series. The list is probably longer than can be taught to many classes and certainly it may be questioned whether some items as, for example, "Discounting notes by banks," should be included. But it must be remembered that the list is merely a tentative list to which other criteria for further reduction are to be applied.

The Materials of Intuitive and Experimental Geometry.

We apply now the same criteria used in the first section in an effort to select the geometry materials. Note that this includes the practice and recall factor. As in the preceding table there is the greatest possible range in grade placement,—from the beginning of the seventh grade to the end of the ninth grade. The list of geometric concepts appearing in four series or more follows:

		A	B	C	D	E	F	G
7, 11	Parallel lines	1, 3	1, 3	1	2	1, 2, 3	1	1
7, 11	Area of triangle	1, 3	1, 2	1	2	1, 2, 3	1	1
7, 11	Square	1, 3	1	1	2	1, 2, 3	1	1, 3
7, 11	Rectangle	1, 3	1	1	2	1, 2, 3	1	1, 3
7, 11	Parallelogram	1, 3	1	1	2	1, 2, 3	1	1, 3
7, 11	Trapezoid	1, 3	1	1	2	1, 2, 3	1	1, 3
7, 11	Area of rectangle	1, 3	1, 2	1	2	1, 2, 3	1	1
7, 11	Area of parallelogram	1, 3	1, 2	1	2	1, 2, 3	1	1
7, 10	Perpendicular lines	1, 3	1	1	2	1, 2, 3	1	1
7, 10	Right angles	1, 3	1, 3	1	2	1, 3	1	1
7, 10	Vertex of angle	1, 3	1, 3	1	2	1, 3	1	1
7, 10	Equilateral triangle	1, 3	1, 3	1	2	2, 3	1	1
7, 10	Right angle triangle	1, 3	3	1	2	2, 3	1	1, 2
7, 10	Area of a trapezoid	1, 3	1, 2	1	2	1, 2, 3	1	1
7, 10	Circumference	1, 3	1, 2	1	2	1, 2	1	1
7, 10	Volume of prism	1	1, 2	1	2	2, 3	2	1, 2
7, 9	Acute angles	1, 3	1, 3	1	2	3	1	1
7, 9	Obtuse angles	1, 3	1, 3	1	2	3	1	1
7, 9	Isosceles triangle	1, 3	3	1	2	2, 3	1	1
7, 9	Hypotenuse	1, 3	3	1	2	2	2	1, 2
7, 9	Area of a circle	1	1, 2	1	2	1, 2	1	1
7, 8	Measurement of angles	3	1, 3	1	2	1	1	1
7, 8	Surface of cylinder	1	1, 2	1	2	2	2	2
7, 8	Volume of cylinder	1	1, 2	1	2	2	2	2
7, 7	Surface of prism	1	1	1	2	2	2	1
7, 7	Volume of pyramid	1	2	1	2	2	2	2
7, 7	Volume of cone	1	2	1	2	2	2	2
6, 10	Straight lines	1, 3	1, 3	1		1, 2, 3	1	1
6, 9	Bisecting lines	1, 3	1, 3		2	2, 3	1	1
6, 9	Bisecting angles	1, 3	1, 3		2	2, 3	1	1
6, 9	Sides of angles	1, 3	1, 3	1		1, 3	1	1
6, 9	Center of circle	1, 3	3	1		1, 3	1	1
6, 9	Arc	1, 3	1	1		1, 2, 3	1	1
6, 9	Definition of cube	1, 3	1		2	2, 3	1, 2	3
6, 8	Supplementary angles	3	3	1	2	1, 2, 3	1	
6, 8	Acute angle triangles	1, 3	3		2	2, 3	1	1
6, 8	Obtuse angle triangles	1, 3	3		2	2, 3	1	1
6, 8	Hexagon	3	1		2	2, 3	1	1, 2
6, 8	Radius	1, 3	1, 3	1		1	1	1
6, 8	Diameter	1, 3	1, 3	1		1	1	1
6, 7	Scalene	3	3	1	2	2, 3	1	
6, 7	Base of a cylinder	1	1, 2	1		2	2	2
6, 7	Attitude of a pyramid	1	2	1	2	2, 3	2	
6, 7	Surface of cone	1		1	2	2	2	2
6, 6	Complementary angles	3	3	1	2	3	1	
6, 6	Surface of pyramid	1		1	2	2	2	2
6, 6	Surface of sphere	1	2	1		2	2	2
5, 8	Perimeter of any polygon	1, 3		1	2	1, 2, 3	1	
5, 7	Volume of a sphere	1	2, 3	1, 2		2		2
5, 6	Line segments		1, 3	1		1	1	1
5, 6	Equiangular triangles	3	3		2	2, 3	1	
5, 6	Rhombus	3		1	2	2, 3		1
5, 6	Area of a square	1	1		2	1, 2		1
5, 6	Pentagon	3			2	2, 3	1	
5, 6	Area of any polygon	1, 3	1	1			1	1
5, 6	Vertex height of a pyramid	1	2	1	2		1, 2	
5, 5	Intersecting lines	3	3		2		1	1
5, 5	Vertical angles	3	3	1	2	3		
5, 5	Altitude of a triangle		3		2	1	1	1
5, 5	Octagon	3			2	2	1	2
5, 5	Lateral faces of prisms		1	1		3	2	2
5, 5	Bases of prisms		1	1		3	2	2
5, 5	Altitude of cylinder	1	1			3	2	2
5, 5	Slant height of a pyramid	1		1	2	2	2	
5, 5	Altitude of a cone	1	2	1		3	2	
5, 5	Center of a sphere	1	2	1		2	2	
5, 5	Similar figures	1	2		2	3	2	
4, 6	Planes	1, 3		1		1	1, 2	
4, 5	Transversal lines	3	3		2	1, 3		
4, 5	Measurement of lines		1, 3	1		1	1	
4, 5	Straight angles	3	1, 3	1	2			
4, 5	Vertex of a triangle	1, 3	3		2		1	

	A	B	C	D	E	F	G
4, 5 Congruent triangles	1, 3	3		3		3	
4, 5 Slant height of cone	1	2			2, 3	2	
4, 4 Adjacent angles	3		1		3	1	
4, 4 Base of a triangle	3	3		2			1
4, 4 Chord			1		3	1	1
4, 4 Concentric circles		1		2	2		1
4, 4 Altitude of a prism			1		2, 3	2	2
4, 4 Radius of a sphere	1	2	1		2		
4, 4 Diameter of a sphere	1	2	1		2		

The table furnishes us 81 items, of which 27 appear in all of the series, and 47 concepts are developed in six of the series. The most striking feature of this table is the large number of concepts concerning which there is an agreement as contrasted with the situation for social-economic forms.

Additional Geometric Materials.

For the sake of completeness it seems worth while to list the additional elements of a geometric nature; for example, the geometric principles and constructions that would meet our criteria. These elements are shown in the following table:

	A	B	C	D	E	F	G
7, 9 Theorem of Pythagoras	3′	2	1	2	2, 3′	2, 3′	2
7, 8 The sums of the angles of a triangle	3′	2, 3	1	2	3′	1	1
6, 7 The base angles of an isosceles triangle are equal	3′	2, 3		3′	3′	1	1
5, 7 If two parallel lines are cut by a transversal, the alternate interior angles are equal	3′	3		2, 3′	3′	3′	
5, 6 Two triangles are congruent if three sides are equal	3′	2, 3		3′	3′	3′	
5, 5 The diagonal of a parallelogram divides it in two congruent triangles	3′	2		1	3′	3′	
5, 5 The opposite sides of a parallelogram are equal and parallel	3′			2	3′	3′	1
4, 5 The diagonals of a parallelogram bisect each other	3′			2	1, 3′	3′	
4, 4 If opposite sides of a figure are parallel and equal, it is a parallelogram	3′	2			3′	3′	
4, 4 If alternate angles are equal, the lines are parallel	3′	3			3′	3′	
4, 4 Two right triangles are congruent if the hypotenuse and an acute angle, etc	3′			3′	3′	3′	
4, 4 Two right triangles are congruent if the hypotenuse and one side, etc	3′			3′	3′	3′	
7, 10 To draw a perpendicular to a point on a line	1, 3′	1	1	2	2, 3	1	1
7, 10 To drop a perpendicular from a point without line	1, 3′	1	1	2	2, 3′	1	1
6, 10 To bisect a line	1, 3′	1, 3		2	2, 3′	1	1
6, 10 To construct an angle equal to a given angle	1, 3′	1, 3	1	2	2, 3′	1	
6, 8 To construct a triangle given three sides	1	2, 3	1	2		1	1
6, 7 To construct parallel lines	1	1	1	2	2, 3′	1	
5, 9 To bisect an angle	1, 3′	1		2	2, 3′	1	1
4, 4 To construct a parallelogram		2		2	2	1	

The table shows fifteen principles and eight constructions. It is important to note that the principles and constructions are listed in order of importance separately. That is to say, a new order begins with the thirteenth element in the above table.

The great variation of materials noted under other sections appears even in the case of geometric principles. The length of the complete list of principles treated in the different books would be astonishing. The third book of Series A contains no less than ten principles that do not appear in any one of the other books. Here it is important to remember that even in the conventional demonstrative geometry courses the field is relatively limited. Thus we note that the list of propositions in plane geometry proposed by the National Committee* is thirty and the number of constructions is seventeen. To be sure, some of the propositions contain two or three parts, but even then the list is not an extended one. One additional fact needs to be emphasized. When these seven series of texts were being constructed the movement for plane geometry in the exploratory course of the junior high school was much stronger than it is at present. It has already been pointed out that the last five series of junior high school texts to be published have not included plane geometry. If we reject the "meaning of a demonstrative proof" from our list of specific objectives, all of the principles, excepting the theorem of Pythagoras and the angle sum rule together with the last two constructions of the preceding table, are ruled out. This can be seen by inspecting the table. When a geometric principle is proved in the demonstrative geometry unit and appears nowhere else in the series, we have indicated it by a 3^1. Thus the preceding list would consist of two geometric principles to be proved by experimental methods and two simple constructions.

The Complete Check of Junior High School Texts.

The foregoing tables have been printed merely to illustrate the technique that will be employed in Chapter V. Here the content of junior high school textbooks is used as one of the five criteria in the final selection of materials. The method illustrated by the preceding pages of this chapter will be used in checking 454 items. The acceptance or rejection of any one of these 454 elements is indicated in Chapter V by the presence or absence of a single symbol.

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*See pages 57-59 "The Reorganization of Mathematics in Secondary Education."

Supplementary Material.

The following material appears in a recent study by Johnson.* In his report he includes six of the seven series of this study. Probably the six books selected from Johnson's table are those included in the writer's study. However, this does not need to be true for we wish merely to present additional evidence on the range and variation of materials on geometric topics.

MODIFIED FROM JOHNSON

Topic	The number of series containing material	Variation in pages	Topic	The number of series containing material	Variation in pages
Area of plane figures	6	10 to 64	Demonstrative Geometry	3	27 to 142
Similar triangles	6	5 to 24	Unit of Trigonometry	5	8 to 24
Volume and Surfaces	6	1 to 55	Total Geometry	Something in each series	86 to 278
Scale drawings	5	3 to 14			

Johnson also submits the following table showing the percentage of the total number of pages in the whole series devoted to geometry material:

AFTER JOHNSON

Series	A	B	C	D	E	F
Total pages in whole series	782	662	679	701	698	822
Percent of Geometry	33	28	30	12	23	15
Percent of Geometry plus Trigonometry	35	32	31	12	27	16

Johnson's tables support our contention that the issues of junior high school mathematics are far from settled. He finds that one author treats volumes and surfaces in a single page whereas another devotes fifty-five pages to this topic. Even if we rule out the one page text as an extreme case, the next two lowest have but ten and eleven pages respectivly. Or take the matter of plane geometry. One of our most gifted and successful authors thinks it takes one hundred forty-two pages to give junior high school pupils the "meaning of a proof." A second series accepts the identical objective in the preface and attempts to achieve it in twenty-seven pages. Which policy is

*John T. Johnson, School Science and Mathematics, June, 1925.

correct? So far as the writer is informed, nobody knows the answer to this question. It seems a reasonably safe guess that both are wrong. At any rate, we are certain that each offering is costing a good many communities considerable sums of money and the reader may be quite certain that in no case is it called experimental.

The material from Johnson is not sufficiently detailed to be of use in Chapter V but it does furnish us, by a general view, a kind of summary of variations existing in whole sections of the junior high school field. The study also shows the per cent of the total pages of each series devoted to geometry and to geometry and trigonometry.

As has been stated in a preceding section, the complete summary of this chapter appears in Chapter V where it may be secured by a detailed study of the long table which appears there.

CHAPTER V

THE OBJECTIVES FOR GRADES SEVEN, EIGHT AND NINE

This chapter will include a list of objectives chosen on the following five criteria:

1. *Objective Studies.*

In a preceding chapter twenty-nine objective studies have been reviewed and a summary table built. Each element considered is checked against this table. If some support in one or more of these objective studies can be found for an element, it is accepted on this basis. This fact will be checked by the symbol "X" in the first column of the table under the heading "S,"—suggesting "scientific study." The absence of a check mark means that no evidence by way of support is known to exist.

2. *Junior High School Texts.*

If an element is developed in four of the seven sets of junior high school texts discussed in Chapter III, it is accepted on this basis. The number of books in which every element is found is indicated in column two under "T,"—suggesting "text." But note that if a given element appears in less than four books it is rejected on this basis.

3. *Practice as Determined by Courses of Study.*

Guiler's outlines for grades seven and eight were included in Chapter II. A similar study revealed the conventional practice for the ninth grade. The combination is accepted as conventional practice. If an item appears in Guiler's lists or in the algebra outline, it is accepted on this basis and an "X" is placed opposite the item in the third column of the table under "P,"—suggesting "practice."

4. *The Outline of the National Committee.*

In the final report, the National Committee on Mathematical Requirements listed the materials for the junior high school grades in considerable detail on pages twenty-one to twenty-eight. It also considered most carefully the aims of

mathematical instruction in Chapter II. Since this report received wide spread discussion and included several significant investigations, mention either direct or by implication in the pages cited is accepted as the fourth criterion. The acceptance of an item will be checked by an "X" in the fourth column of the table under "N,"—suggesting "National Committee."

5. *The Competent Jury.*

An extensive list of 520 objectives was submitted to a jury of ten; five of these were leaders in the teaching of mathematics and five were prominent men in general education. Each judge was asked to accept or reject each item in a letter which included the following directions: "Will you read each of the four sets of objectives carefully and strike out each one of the elements which you would not hold as a worthy pupil outcome in junior high school mathematics? Keep in mind the following:

(1) The objectives as set forth in Cardinal Principles of Secondary Education, and

(2) Assume that you have a son or daughter with an I. Q. of ninety or higher passing through grades seven, eight and nine. As you read each item in turn, weigh it in the balance, then either strike it out or let it stand unmarked as accepted. When you have finished, you may wish to add items that seem to have been omitted from my list."

Each member of the jury was chosen after most careful thought. Only one substitution had to be made. That is to say, one member in the original list refused to serve "because the work was too much like Charters'" and this he considered absurd. The complete list is as follows:

Mathematics Leaders:

Charles M. Austin, First President, National Council of Mathematics Teachers.

Dean J. H. Minnick, University of Pennsylvania, Second President, National Council of Mathematics Teachers.

Clifford B. Upton, Associate Professor, Teachers College, Columbia University.

Dr. John R. Clark, Editor of the Mathematics Teacher.

William Betz, Supervisor of Mathematics, Rochester, New York.

General Educators.

Thomas H. Briggs, Professor of Education, Teachers College.

Calvin O. Davis, Professor of Secondary Education; University of Michigan.

Dr. Charles L. Spain, Deputy Superintendent of Schools, Detroit.

Stuart A. Courtis, Professor of Education, University of Michigan.

Clifford Woody, Head of Department of Educational Psychology, University of Michigan.

Any item that receives a vote of acceptance from three members in each of the two groups is considered acceptable on this basis. In any event, the vote on each item will be found in the fifth column under the heading "J,"—suggesting "jury."

The "3" in "32" means that 3 out of 5 mathematics judges approved the element in question, while the "2" in "32" means that 2 out of the 5 general educators accepted the item. A "55" means that each group of 5 passed the element. Note that in every case the first figure refers to the mathematics judges and the last figure to the general educators.

Final Acceptance or Rejection.

Any item which is accepted on the basis of any three criteria is included in the basic list under each of our four groups,—Attitudes, Concepts, Information and Skills. All other items are included in the subsidiary list.

The table which follows includes the list of attitudes:

SECTION A—ATTITUDES

Basic List

The table which follows includes the list of attitudes:—

	S	T	P	N	J
1. Assuming responsibility for correct results*		5	X	X	55
2. Recognizing importance of an "estimated answer" as a control in the computation of a problem	X	4		X	55
3. Appreciating the use of geometry in art		6		X	55
4. Appreciating the use of geometry in architecture		4		X	55
5. Being interested in and constantly scrutinizing personal growth in the skills of mathematics		4			55
6. Appreciating necessity for desirable habits of thrift		4		X	55
7. Being critical of investments		5		X	55
8. Appreciating the power of compound interest		4	X	X	55
9. Being critical of the extent to which a computed result based on measurement is accurate		4		X	55

*Items 1 and 2 assume the ability to check and the ability to estimate.

SUBSIDIARY LIST

1. Caring for the beautiful geometric forms in nature	0	X	55
2. Appreciating the need for the systematization of data in problem solving	0	X	45
3. Appreciating the value of scrutinizing data to seek relations in problem solving	0	X	55
4. Desiring precision of statement	1	X	55
5. Discrimination between the true and the false	0	X	55
6. Seeking for continuity of law	0	X	34
7. Desiring to analyze a complex situation into simpler parts	0	X	45
8. Rejecting irrelevant factors in a problem	0	X	55
9. Desiring to discover the general law when examining particulars	0	X	55
10. Desiring to get "at the bottom" of a situation,—the attitude of enquiry	0	X	55
11. Appreciating the value of organization as an aid to memory and understanding	1	55	..
12. Being annoyed with vagueness and incompleteness	0	X	55
13. Loving thoroughness and clearness	1	X	55
14. Appreciating the value of formulating hypotheses in problem solving	1		55
15. Appreciating the need for recalling past experience in the solution of a problem—the desire for fertility of suggestion	0		55
16. Appreciating the value of having suggested solutions to a problem "pass in review" and for evaluating each in turn	0		55
17. Wishing to cultivate deferred judgment in dealing with a problem or desiring to seek new evidence that may modify a definition, law or concept,—the wish to be openminded	0		55
18. Appreciation of the contribution of mathematics to the progress of civilization	0	X	*

COMMENTS ON SECTION A

1. Both types of judges rate these attitudes very high. It is practically certain that these judgments were more carefully considered than on any other list.

2. Assuming that these attitudes are desirable the text books include little evidence of a conscious drive to secure these outcomes. Probably the appreciational and disciplinary values are clear in the minds of the authors but they fail to get these into books for children. If it is expected that these deeper values will be by-products of symbolic manipulation then we are assuming a fallacy.

3. The makers of courses of study either do not think these attitudes are worth while or else they are assuming that such outcomes are achieved without special efforts. Practice puts the emphasis upon information and skills,—not upon attitudes and concepts.

*Suggested by two judges, one mathematics teacher and one general educator.

4. The available objective studies throw little light on this phase of the problem. The formal aspects are the easy things to measure and to investigate.

5. It should not be assumed that because an attitude appears in the subsidiary list that it is undesirable,—we have had such extensive changes in the teaching of mathematics in the last decade that we need to keep the door to further progress open.

SECTION B—CONCEPTS

Basic List

Through numerous concrete illustrations the pupil should be given an understanding of the following concepts:

	S	T	P	N	J
1. Ratio	X	7	X	X	55
2. Proportion	X	7	X	X	55
3. Positive and negative numbers	X	7	X	X	55
4. Length	X	7	X	X	55
5. Area	X	7	X	X	55
6. Volume	X	7	X	X	55
7. A surface	X	7	X	X	55
8. An equation	X	7	X	X	55
9. A formula	X	7	X	X	55
10. A graph	X	7	X	X	55
11. An algebraic number	X	7	X	X	54
12. A coefficient	X	7	X	X	55
13. An exponent	X	7	X	X	55
14. A root	X	7	X	X	55
15. A power	X	7	X	X	55
16. Perpendicular lines	X	7	X	X	55
17. A triangle	X	7	X	X	55
18. A square	X	7	X	X	55
19. A rectangle	X	7	X	X	55
20. A parallelogram	X	7	X	X	55
21. A cube	X	7	X	X	55
22. A rectangular prism	X	7	X	X	53
23. A cylinder	X	7	X	X	55
24. A factor (algebraic)	X	7	X	X	45
25. A product (algebraic)	X	7	X	X	55
26. A fraction (algebraic)	X	7	X	X	55
27. Pi	X	7	X	X	55
28. Parallel lines	X	7	X	X	55
29. A solution of an equation	X	7	X	X	55
30. Net price	X	6	X	X	54
31. Simple interest	X	7	X	X	55
32. Profit	X	7	X	X	55
33. Loss	X	7	X	X	55
34. Taxation	X	7	X	X	55
35. A root of an equation	X	7	X	X	53

	S	T	P	N	J
36. Graphic representation	X	7	X	X	55
37. Insurance	X	7	X	X	55
38. Equality	X	5	X	X	55
39. Similarity	X	7		X	54
40. A measurement	X	7		X	55
41. Statistical graph	X	7		X	55
42. A drawing to scale	X	6		X	55
43. Variation	X	4		X	43
44. An angle	X	7		X	55
45. A circle	X	7	X	X	55
46. A quadratic		7	X	X	54
47. A pyramid		7		X	55
48. An approximation in a measurement	X	4		X	55
49. Compound interest		4	X	X	54
50. A perimeter		7	X	X	55
51. Dependence	X	4		X	54
52. Congruence		4		X	53
53. A trapezoid		6		X	53
54. An error in a measurement	X	3		X	55
55. An average	X	6		X	55
56. A cone		7		X	55
57. A sphere		7		X	55
58. An indirect measurement		7		X	55
59. A vertex		7		X	55
60. A tangent of an angle		6		X	55
61. A sine of an angle		6		X	34
62. A cosine of an angle		6		X	33
63. Algebra as a tool of science	X			X	55

SUBSIDIARY LIST OF CONCEPTS

	S	T	P	N	J
1. Symmetry		2		X	55
2. A frequency distribution		2		X	35
3. Law of order in multiplication		6		X	42
4. Law of order in addition		4		X	42
5. A median		1		X	34
6. A mode		1		X	24
7. Central tendency of a distribution		1		X	35
8. A deductive proof		4		X	20
9. A logarithm		2		X	42
10. A locus		2		X	34
11. A maximum		0			45
12. A minimum		0			45
13. The standardization of a measuring unit		2			55
14. Inequality		2			22
15. An aliquot part		6			33
16. Direction		2			55
17. Rate		7			55
18. Weight		3			55
19. Velocity		4			43

COMMENTS ON SECTION B

1. In this part the judgments of leaders tend to parallel material in texts; it will be recalled that this was not at all the case in the list for attitudes.

2. The courses of study are not written to indicate what concepts are definitely desired. It is inconceivable that some of these fundamental concepts are not hoped for or desired by the makers of courses of study.

SECTION C—ABILITIES

Basic List

Fundamental Processes (as applied in arithmetic)

	S	T	P	N	J
1. To apply the fundamental operations of arithmetic to whole numbers with perfect accuracy*	X	7	X	X	55
2. To apply the fundamentals of arithmetic to decimal fractions having less than 5 digits	X	5	X	X	45
3. To apply the fundamental operations of arithmetic to fractions having denominators 2, 3, 4, 5, 6, 8, 10, 12, and 16 with perfect accuracy		6	X		45
4. To express a ratio as a decimal	X	5	X	X	55
5. To express a ratio as a per cent	X	6	X	X	55
6. To use the short cut in multiplying a number by 10	X	7		X	55
7. To use the short cut in multiplying a number by 25	X	7		X	45
8. To use the short cut in multiplying a number by 50	X	7		X	45
9. To use the short cut in multiplying a number by 100	X	6		X	55
10. To use the short cut in multiplying a number by 1000	X	6		X	55
11. To use the short cut in dividing a number by 10	X	7		X	55
12. To use the short cut in dividing a number by 25	X	6		X	34
13. To use the short cut in dividing a number by 50	X	6		X	35
14. To use the short cut in dividing a number by 100	X	7		X	55
15. To use the short cut in dividing a number by 1000	X	7		X	53
16. To find the square root of an arithmetic number by a table	X	4		X	55
17. To express these common fractions in their decimal and per cent equivalents: ½, ⅓, ⅔, ¼, ¾, ⅛, ⅜, ⅝, ⅞, 1/16, 2/5, ⅗, ⅘, 1/6	X	3		X	45

*Allowing only an occasional slip.

Per Cents

	S	T	P	N	J
18. To find any per cent of a number	X	7	X	X	55
19. To find the net price of an article on which two discounts are allowed	X	7	X	X	44
20. To compute per cent of increase	X	7	X	X	55
21. To compute per cent of decrease	X	7	X	X	55
22. To find what per cent one number is of another		7	X	X	55
23. To find a number when a certain per cent of it is known		6	X	X	54

Graphs

24. To interpret a bar graph	X	6		X	55
25. To make a bar graph	X	7		X	55
26. To interpret a line graph	X	6		X	55
27. To make a line graph	X	7		X	55
28. To interpret a circle graph	X	5		X	55
29. To make a circle graph	X	6		X	55
30. To interpret the graph of a formula	X	4		X	54
31. To graph a formula with two unknowns (linear)— difficulty not to exceed $F = 9/5\ C + 32$		5	X	X	44
32. To use cross section paper to picture amounts of things	X	4		X	55
33. To express a ratio graphically	X	7		X	35

Statistics

34. To find the average of a group of arithmetic numbers	X	5		X	55
35. To read a statistical table as illustrated by a height and weight table, a simple interest table, or a compound interest table	X	7		X	55

Geometry

36. To interpret newspapers and magazine references to metric units of length, area and volume	X	4	X	X	45
37. To read an angle that is lettered in one of the three conventional ways	X	7		X	55
38. To letter an angle in one of the conventional ways	X	7		X	55
39. To measure a line (segment) with a ruler accurate to the nearest tenth of an inch	X	4		X	55

	S	T	P	N	J
40. To use a scale drawing in the plan of a building or the reading of a map	X	6		X	55
41. To draw a triangle to scale when sufficient parts are known	X	5		X	55
42. To bisect an angle with the compasses	X	6		X	55
43. To bisect a line (segment) with the compasses	X	6		X	55
44. To draw a right angle with a ruler	X	4		X	55
45. To draw a circle with the compasses	X	7		X	55
46. To draw parallel lines with the protractor	X	3		X	55
47. To measure an angle with a protractor		7		X	55
48. To draw an angle of required size with a protractor		7		X	55
49. To compute one angle of a triangle when two angles are known		6		X	54
50. To measure an inaccessible distance by a scale drawing when necessary data are given		6		X	55
51. To use proportion in similar triangles so as to compute a side when the necessary data are given		7		X	55
52. To construct a hexagon by means of a circle		6		X	44
53. To construct a line perpendicular to another line with the compasses		7		X	45
54. To draw a right angle with the compasses		7		X	45
55. To draw parallel lines with the compasses		6		X	55
56. To draw an equilateral triangle (isosceles*)		7		X	55
57. How to use the 30-60 right triangle in drawing angles	X			X	44
58. How to use the 45-45 right triangle in drawing angles	X			X	45

*Added as a special case.

Business Forms and Devices

59. How to check a bill (e. g. grocery bill)	X	4	X	X	55
60. To find the maturity date of a note		4	X	X	55
61. To find the time between the date when a note is made and the date when it is paid		6	X	X	45
62. To read market quotation of stocks	X	4		X	55
63. To read market quotation of bonds	X	3		X	55
64. To read an electric meter	X	4		X	35
65. To read a gas meter	X	4		X	35
66. To find the time between dates by an "interest day table"	X	1		X	45
67. To discount a note		5	X		43
68. To find the balance of a savings account		6	X		55

FUNDAMENTAL PROCESSES (as applied to algebraic numbers)

69. To apply fundamental operations to positive and negative numbers. (No literal numbers involved)	X	7	X	X	55
70. To apply fundamental operations to positive and negative numbers (algebraic)	X	7	X	X	55
71. To multiply a monomial by a monomial	X	7	X	X	55
72. To multiply a polynomial by a monomial	X	7	X	X	55
73. To divide a monomial by a monomial	X	7	X	X	55
74. To divide a polynomial by a monomial	X	7	X	X	55
75. To use the law of exponents in multiplication: $X^m X^n = X^{m+n}$	X	7	X	X	55
76. To use the law of exponents in division: $X^m \div X^n = X^{m-n}$	X	7	X	X	55
77. To apply fundamental operations to algebraic fractions of the type of $\frac{a}{b}$	X	7	X	X	55
78. To multiply a polynomial by a polynomial		7	X	X	34
79. To divide a polynomial by a polynomial		7	X	X	34
80. To find the square root of an artithmetic number by the traditional algebraic method		6	X		34
81. To find the square root of an algebraic number		4	X		35
82. To apply the fundamental operations to algebraic fractions of the type $\frac{a+b}{c+d}$		7	X	X	35

SPECIAL PRODUCTS AND FACTORS†

83. To give at sight special products of the type: $(ax+b)(cx+d)$		6	X	X	23
84. To give at sight special products of the type: $(ax+cy)^2$		7	X	X	32
85. To give at sight special products of the type: $(x-y)(x+y)$		7	X	X	34
86. To give at sight the factors of an algebraic number of the type: ax^2+bx+c		7	X	X	23
87. To give at sight the factors of an algebraic number of the type: $a^2+2ab+b^2$		7	X	X	34
88. To give at sight the factors of an algebraic number of the type: $a^2-2ab+b^2$		7	X	X	34
89. To give at sight the factors of an algebraic number of the type: x^2-y^2		7	X	X	44

†Probably the monomial factor should have been treated as a separate case. But this case would surely be treated if anything at all were done with the topic of factoring and special products. The more difficult cases were not submitted to the judges because the sources used for the original list did not supply these. What would have happened to the more complex types of factoring is clear. Note that even the mathematics judges are becoming severely critical at this point of the table.

TRANSFORMATIONS‡

90. To transform: $\sqrt{a^2b} = a\sqrt{b}$	X	6	X	X	44
91. To transform: $\sqrt{\frac{a}{b}} = \frac{\sqrt{ab}}{b}$	X	6	X	X	44
92. To transform: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$	X	3	X	X	44
93. To transform: $X^{\frac{1}{2}} = \sqrt{X}$	X	2	X	X	34

‡The writer assumes that these transformations will be considered as types. Thus in number 93, the teacher surely would desire to develop the concept of a fractional exponent. This could be defended on practical grounds, as for example, the use in the development of logarithms and the application to the slide rule. But the chief reason is to furnish a basis for the extension of the pupil's concept of an exponent.

FORMULAS

94. To use the formular $A = bh$ (In computing the area of a rectangle)	X	7	X	X	54
95. To use the formula $A = s^2$ (In computing the are of a square)	X	7	X	X	54
96. To use the formula $A = \pi r^2$ (In computing the area of a circle)	X	7	X	X	54
97. To use the formula $C = \pi d$ (In computing the circumference of a circle)	X	7	X	X	54
98. To use the formula $V = luh$ (In computing the values of a rectangular prism)	X	7	X	X	52
99. To use the formula $V = s^3$ (In computing the value of a cube	X	5	X	X	54
100. To use the formula $V = \pi r^2 h$ (In computing the value of a cylinder)	X	7	X	X	53
101. To use the formula $I = prt$ (In computing interest)	X	7	X	X	54
102. To use the formula $A = \frac{bh}{2}$ (In computing the area of a parallelogram		7	X	X	54
103. To use the formula $V = \frac{bh}{3}$ (In computing the value of a pyramid		7	X	X	53
104. To use the formula $S = \pi r^2 + 2\pi rh$ (In computing the surface of a cylinder)		6		X	43
105. To solve any formula in (94-104) for any letter by a proper use of the axioms	X	5	X	X	45
106. To evaluate any formula in (94-104) for any letter having given the values of all other letters that occur in that formula	X	6	X	X	54
107. To translate a practical rule of procedure in arithmetic by a formula (difficulty not to exceed $F = \frac{9}{5}c$ plus 32)		4	X	X	45

	S	T	P	N	J
108. To solve the formula $a^2+b^2=c^2$ for either a or b		7	X	X	44
109. To use the formula $V=\frac{4}{3}\pi r^3$ (In computing the volume of a sphere)		7		X	53
110. To use the formula $A=\frac{h(B+b)}{2}$ (In computing the area of a trapezoid		7		X	53
111. To translate a formula (difficulty not to exceed $F=\frac{9}{5}C+32$)	X	5	X	X	55
112. To express by an equation between two numbers that is stated in the form of an English sentence. (Difficulty not to exceed "To find the Fahrenheit reading corresponding to some known Centigrade reading multiply it by $\frac{9}{5}$ and add 32")	X	7	X	X	54
113. Ability to find the value of the unknown number in a formula with two unknowns having given the value of the other unknown—difficulty not to exceed $F=\frac{9}{5}C+32$		6	X	X	54
114. To find the third side of a right angle triangle by the formula $a^2+b^2=c^2$ when two sides are known		6	X	X	54

EQUATIONS

115. To use the addition, subtraction, multiplication and division axioms in the solution of an algebraic equation	X	7	X	X	55
116. To clear an equation of fractions, the denominators to be limited to arithmetic numbers, the least common multiple to be found by inspection	X	7	X	X	55
117. To interpret a negative number as an answer when the unknown forms one member of the equation	X	7	X	X	54
118. To find the fourth number in a proportion when three numbers are given	X	7	X	X	55
119. To clear an equation of fractions, the last common multiple to be limited to an algebriac number that may be found by inspection and to involve only the cases of factoring listed above		7	X	X	45
120. To solve similtaneous linear equations in two unknowns by the addition and subtraction method		7	X	X	44
121. To solve an equation in one unknown containing fractions with binomial denominator,—not to exceed in difficulty $\frac{1}{2}+\frac{3}{X+2}=4$		7	X	X	44
122. To solve simultaneous linear equations in two unknowns by the graphic method		6	X	X	34

Trigonometry

123. To read a table of tangents with three decimal places	X	6		X	43
124. To apply the tangent relation in solving a right angle triangle	X	7		X	44
125. To apply the sine relation in solving a right angle triangle		6		X	43
126. To apply the cosine relation in solving a right angle triangle		6		X	43
127. To find a side of a triangle by the tangent relation		7		X	83

SUBSIDIARY LIST

Fundamentals

1. How to remove parentheses when a mathematical statement has two—one within the other		4	X		25
2. To use the short cut in multiplying a number by $12\frac{1}{2}$		6			34
3. To use the short cut in multiplying a number by $33\frac{1}{3}$		6			44
4. To use the short cut in multiplying a number by $66\frac{2}{3}$		6			24
5. To use the short cut in multiplying a number by 75		6			34
6. To find the square root of an arithmetic number by the "estimate" method	X	1			44
7. How to use abbreviated multiplication					44
8. How to operate a slide rule					34
9. To compute with decimal coefficients		1		X	52

Graphs

10. To express direct variation by a graph	X	2		X	52
11. To read a compound interest graph		1			53

	S	T	P	N	J
12. To solve a quadratic equation by the graphic method		5			13
13. To find the square root of arithmetic numbers by means of graph		1			32
14. To express inverse variation with a graph		0		X	32

Per Cents

15. To find the net price of an article on which three discounts are allowed		4			33

Statistics

16. To find the average of the scores in a distribution table		2		X	34
17. To find the mode of the scores in a distribution table		1		X	23
18. To find the median of the scores in a distribution table		1		X	54

On Problem Solving

19. To recognize missing data in a problem		0			55
20. To recognize non-essential data in a problem		0			55

On Reading Scales

21. How to read a micrometer		0			14

Formulas

22. To use the formula $V = \pi\frac{r^2h}{3}$ (In computing the volume of a cone)	7		X	52
23. To use the formula $S = 4\pi r^2$ (In computing the surface of a sphere)	7		X	52
24. To write a formula showing how two related numbers change having given a table of paired values	0		X	54
25. To extend a table showing how two related numbers change	0			44
26. To formulate an arithmetic rule when inspecting a table showing how two related numbers change	0		X	55

Equations

27. To solve a quadratic equation by the factoring method	7	X		23
28. To solve a quadratic equation by the completing square method	7	X		24
29. To solve a quadratic equation by the formula method	6	X		32
30. To solve simultaneous linear equations in two unknowns by the substitution method	6	X		24
31. To solve simultaneous equations in two unknowns by the comparison method	3	X		14
32. To solve simultaneous linear equations in three unknowns by the addition and subtraction method	1	X		32
33. To study the change in the value of a quadratic expression in one unknown as the unknown is assigned different values	2		X	14

Transformations

34. To transform: $X^{1/3} = \sqrt[3]{X}$	2	X		34
35. To recall automatically the value of $\sqrt{2}$, $\sqrt{3}$, and $\sqrt{5}$	3			21
36. To transform: $\frac{X^{-m}}{X^{-n}} = \frac{X^n}{X^m}$ (m and n not beyond 6)	2	X	X	24

Geometry

37. To measure the height of an object by shadow reckoning	7			55
38. To approximate a distance by the use of a pace scale	1			55
39. To estimate the area of a figure drawn on cross section paper	3			54
40. How to add segments				44
41. To measure a line (segment) with compasses accurate to the nearest tenth of an inch	1			45
42. To construct a square	5			52
43. To construct an octagon by means of a circle	3			42
44. To test a triangle to see if it is a right triangle having given the length of the three sides	3			55
45. To demonstrate the theorem of Pythagoras. (This assumes from 8 to 12 weeks of preparatory work in plane geometry of the traditional type. The argument for this is that every boy or girl ought to learn before leaving the school how to prove a statement.)	4			14

Trigonometry

46. To find a table of tangents experimentally		4	X	23

COMMENTS ON SECTION C

1. The most outstanding fact in this section is the attitude of the mathematical judges on manipulation of symbolism. Only two would have pupils factor at sight an algebraic number of the type of ax^2+bx+c; only three would have pupils give the factors of a squared trinomial; only two would give at sight special products of the type $(ax+b)\ (cx+d)$; and only two would have pupils remove parentheses when a mathematical statement has two,—one within the other. It is to be noted that the jury of general educators is far less critical of abstract manipulation. An inspection of this table of skills shows that the mathematical group would decrease manipulation in startling fashion.

2. Note that the list of formulas accepted on this basis agrees to a marked extent with the list of formulas accepted on an earlier criterion (inclusion in junior high school texts.)

3. The number of short-cuts is far lower than one would have expected on this basis, but the choice could be supported by excellent authority. The difficulty with short-cuts is that so few pupils develop enough skill to make these methods of practical value. Few adults in ordinary affairs use the short-cuts taught in texts and certainly not many textbooks give sufficient drill to fix the use of short-cuts as permanent habits.

4. It will be noted that in the early part of each major section the agreements between the mathematics judges and the general educators is very close. The cause is not difficult to find. Since the general educators are cautious their vote tends to be high in all cases and hence, wherever the mathematics jury give a high vote to an element the result is agreement.

SECTION D—INFORMATION

Basic List

Fundamental Processes

	S	T	P	N	J
1. How to read numbers up to billions	X	4			55
2. Meaning of correct to n decimal places		4		X	55
3. Some activities of modern life require a high degree of accuracy of measurement	X	0		X	55

Per Cents

4. Per cents means hundredths	X	7	X	X	55
5. How to compute profit on cost	X	7	X	X	55
6. How to compute profit on selling price	X	7	X	X	55
7. Interest = principal x rate x time	X	6	X	X	55
8. Meaning of commission		7	X		55

Arithmetic of the Community

9. These elements in* insurance: fire insurance, life insurance, casualty, face, term, beneficiary, adjusted ordinary life, limited-payment life, endorsement, protection, expectation of life, dividend, rates, (policy**, premium, endowment)		7	X	X	54
10. How change is commonly made in the business world	X	4			55

*Not all texts mention all the elements; the report of National Committee mentions merely "insurance."

**Added as being implied.

Arithmetic of the Home

11. The elements of a personal account	X	7	X	X	55
12. The elements of a household account	X	7		X	55
13. The general facts of the personal and family budget system	X	4	X	X	55
14. How to find cost of sending parcel by parcel post		4	X	X	55

Arithmetic of Investments

	S	T	P	N	J
15. The elements of a bank account	X	6	X	X	55
16. The elements of a check	X	7	X	X	55
17. The methods of endorsing a check	X	7	X	X	55
18. The elements of a receipt	X	5	X	X	55
19. The computation of simple interest on a promissory note		7	X	X	55
20. The methods of endorsing a promissory note		7	X	X	55
21. The elements of a promissory note		7	X	X	55
22. These general elements about bonds:* maturity, coupon, par value, Liberty Bond, municipal bond, and pike bond		6	X	X	54
23. These general elements about stocks:* dividend, corporation, board of director, par value, rate of dividend, broker, brokerage, rate of income, common stock, preferred stock		6	X	X	54
24. Meaning of certificate of stock		6	X	X	55
25. The principles involved in buying on the installment plan	X	2	X		55
26. What a mortgage is		5	X		54
27. Meaning of overhead	X	5			45

*Not all texts mentioned all items, the report of National Committee mentioned only "notion of stocks and bonds."

ARITHMETIC OF BANKING

28. The elements of a savings account	X	7	X	X	55
29. Meaning of amount (in compound interest)		7	X	X	54
30. The elements of a money order		3	X	X	55
31. The elements of a bank draft		4	X		55
32. Meaning of bank discount		4	X		54

GEOMETRY*

	S	T	P	N	J
33. Meaning of radius	X	7	X	X	55
34. Meaning of perimeter	X	7	X	X	55
35. Meaning of perpendicular	X	7	X	X	55
36. Meaning of diameter	X	7		X	55
37. Meaning of arc	X	7		X	55
38. Meaning of right angle	X	7		X	55
39. Meaning of altitude (e. g. of a triangle)	X	7		X	55
40. Meaning of obtuse angle	X	7		X	55
41. π is approximately 3 1/7 or 3.14	X	7		X	55
42. Meaning of acute angle	X	7		X	55
43. Meaning of horizontal axis	X	7		X	55
44. Meaning of vertical axis	X	7		X	55
45. To locate a place horizontal and vertical axes may be used	X	7		X	55
46. Meaning of isosceles triangle		7		X	55
47. Meaning of equilateral triangle		7		X	55
48. Meaning of polygon		7		X	55
49. The sum of the angles of a triangle is 180°		7		X	55
50. The theorem of Pythagoras		7		X	54
51. Meaning of face (of solid)		7		X	55
52. Meaning of corresponding sides		7		X	55
53. If two triangles are similar, then the ratio of any two corresponding sides is equal to the ratio of any other pair of corresponding sides		6		X	44
54. Meaning of transversal	X	5			54
55. Meaning of corresponding angles		5		X	45
56. Two triangles are similar if two angles of the first triangle are equal respectively to the two angles of the second triangle		5		X	44
57. Two triangles have the same size and shape if two angles and the included side of one equal respectively the two angles and the included side of the other		4		X	14
58. Two triangles have the same size and shape if two sides and the included angle of the first triangle are equal respectively to two angles and the included side of the second triangle		4		X	44

*The agreement of the two sets of juries on these geometric items is astonishing for two reasons (1) the material was not taught in the grades when these educators went to school, and (2) the material does not appear in the conventional and widely used texts. If the reader thinks the juries were merely careless or fatigued then he should turn to page 115 and note the extent to which the judges rejected very familiar items of arithmetic.

Algebra

59. How to change form of algebraic fraction without changing the value	X	7	X	X	55
60. Meaning of unknown	X	7	X	X	54
61. Meaning of binomial	X	7	X	X	55
62. Meaning of opposite quantities	X	7	X	X	45
63. Meaning of members of an equation	X	7	X	X	55
64. Meaning of base (distinct from power)	X	6	X	X	55
65. Meaning of radical		7	X	X	55
66. Meaning of trinomial		7	X	X	55
67. Meaning of constant	X	5	X	X	55
68. Meaning of variable	X	6	X	X	54
69. Meaning of cross products		4	X	X	43
70. Meaning of degree of a number		4	X	X	44
71. Meaning of degree of an equation		4	X	X	44
72. Meaning of degree of a term		4	X	X	44
73. Meaning of quadratic trinomial		4	X	X	31
74. Meaning of quadratic trinomial square		4	X	X	21
75. Meaning of cube root		3	X	X	53
76. Meaning of like terms	X	5	X		55
77. Meaning of linear function	X	4		X	43
78. Meaning of descending powers		3	X	X	55
79. Meaning of highest common factor		1	X	X	45
80. Meaning of reciprocal		5	X		54
81. Meaning of satisfy		7	X		45
82. Meaning of similar terms		5	X		55
83. Meaning of identity		4	X	X	54
84. Meaning of partial products		6	X	X	55
85. Meaning of trinomial square		5	X	X	43
86. Meaning of polynomial		7	X		45
87. Meaning of origin		7	X		54
88. Meaning of means (in proportion)		5	X		45
89. Meaning of least common multiple		4	X		45
90. Meaning of dependent variable		4		X	43
91. Meaning of extremes (in a proportion)		5	X		45

Trigonometry

92. Meaning of trigonometry	X	6		X	55

Units of Measure*

To know the sources of information concerning the relations between units of measure in each of the following groups:

93. Mill, cent, dollar, eagle	X	7	X		*

*To the judges these units together with many others were submitted in the list to be taught on a "skill level." Six of the judges suggested that this be placed on the information level. It is to be regretted that the opinions of the judges cannot be furnished. Enough, however, approved these given on the "skill level" that we may safely assume that all would be approved by at least a majority on the "information" level.

Geometry				
51. Meaning of straight angle	6			55
52. Meaning of vertical angles	7			55
53. Meaning of parallelepiped	3			52
54. Meaning of octagon	5			55
55. Meaning of zero of a scale	3			55
56. To locate a place, its distance and direction from some reference point may be used	0		X	55
57. Man has invented fine instruments for measuring angles, (e. g. transit, compass)	5			55
58. Meaning of angle of elevation	5			54
59. Meaning of angle of depression	4			54
60. Meaning of central angle	3			54
61. Meaning of subscript	3			44
62. Two triangles are similar if the ratio of one pair of corresponding sides equals the ratio of another pair of corresponding sides, and if the included angles are equal	1		X	23
63. Meaning of rhombus	6			51
64. Meaning of sector	2			55
65. Meaning of directed number	2		X	43
66. Meaning of reflex angle	3			53
67. Simple uses of a T square			X	55
68. Simple uses of a plumb line				45
69. Important parts of a transit	4			24
70. Important parts of a compass	0			45
71. Meaning of turning tendency	0			14
72. Meaning of a section	2			55
Algebra				
73. Meaning of independent equations	3	X		23
74. Meaning of contradictory equations	4	X		23
75. Meaning of rational	2	X		43
76. Meaning of equivalent equations	2	X		32
77. Meaning of prime	6			45
78. Meaning of probability	0			11
79. Meaning of arithmetic progression	1			21
80. Double use of plus and minus signs	6			35
81. Meaning of geometric progression	1			14
82. Meaning of quadratic function	2			21
83. Meaning of quadratic surd	3			11
84. Meaning of dependent equations	2			23
85. Meaning of mantissa	1			32
86. Meaning of order of radical	1			14
87. Meaning of joint variation	2			03
88. Meaning of mixed expression	3			45
Logarithms				
89. Meaning of characteristic	2			22

Findings and Comments:

1. In the basic course of study there are nine attitudes, 63 concepts, 127 abilities and 106 items of information. The total list of elements in the basic list is 305.

In the subsidiary list we find 18 attitudes, 19 concepts, 46 abilities and 66 items of information, totalling 149. Altogether 454 elements have been considered in this study.

In the following discussion our attention will be directed to the basic list of 305 elements.

2. The basic list represents a course of study that is *definite.* It includes scores of items on which many state courses of study are non-committal. The beginning teacher especially is desirous of knowing whether or not she is expected to teach a given task. Previous to this study the closest approximation to a definite course was to be found in Guiler's dissertation.*

The first objection to Guiler's objectives is that the method is fundamentally not sound. All who are thoughtful about school affairs know that an enormous amount of material has accumulated in our textbooks and in our courses of study that is obsolete and some of which never was of any use to any considerable part of our citizens. It is futile to inventory what we have and accept what we find as our goals in education. The method does not offer a chance for growth. Changing conditions demand new materials of instruction. Guiler's method practically guarantees that there will be no *progress.* On the other hand his method insures that we will hold on to topics and mathematical processes long after they have ceased to serve. The evidence on this last is clear. Consider the fact that Guiler includes in his minimum essentials for the seventh grade the "ability to find bank discount," the "ability to compute compound discount," and that he includes in his list of minimum essentials for the eighth grade "ability to measure and estimate surfaces for purposes of making computations for plastering, painting, and *carpeting.*"

It is astonishing that Guiler did not become suspicious of his "drag net" when it brought up carpeting. The evidence is clear that Guiler could have had a more reliable list of specific objectives by using the judgment of two or three leading writers of arithmetic in the field. It is inconceivable that any one of the authors of the recent books in wide use would have included some of the things in Guiler's list, not to mention the many fine things omitted. The second objection is that his list while seeming to be very definite is in actual fact too vague to be of any help to the classroom teacher. Consider for example his objective "ability to compute compound inter-

*A doctor's dissertation filed at the University of Chicago. Guiler's method is given on page 79

est." If this really means what it says, it cannot be defended. Children in the seventh grade can learn in a general way that there is such a thing as compound interest and a few other facts involved. In short, we can teach the topic on an appreciational level, but it cannot be placed and is not in fact taught on a *skill level.* There are other topics that are vague and deceptive.

3. The basic list represents *growth.* It includes many of the new things that have been advocated in the reorganization movement. At any rate the basic list does not include plastering, painting and carpeting problems as does the course of study by Guiler.* Such problems receive so low a rating that they are not even suggested in our subsidiary list.

4. The basic list has been constructed in the light of existing objective evidence. Guiler insists, and he is entirely correct, that no existing study furnishes a basis for the selection of materials, but there is no reason why all objective studies should be neglected in the selection of materials. The objective evidence has been stretched as far as it will go in this investigation.

5. The new materials suggested by books recently published for these grades have been given due weight. The men and women writing these books have given much thought to the problem. It does not seem that a course of study should be built by neglecting the new books in the field. In fact, it would not be practical nor feasible to suggest a course of study for which there exist no teaching materials.

One cannot read Bobbitt's materials without entertaining the fear that the teachers would be hopelessly confused. The teacher may face the practical problem of teaching one hundred or two hundred pupils. A vague philosophy will not be sufficient to meet her needs.

6. The proposed course of study has given weight to practice on the assumption that things worth while tend to persist in courses of study, but it is given weight only to the extent of one out of five votes. This is the essential difference between the proposed basic list and Guiler's.

7. The course of study recognized the great value of the outline growing out of the nation-wide discussion stimulated by the National Committee on Mathematical Requirements. For a decade at least, any intelligent effort to build a course

*Guiler's method is described on page oo.

of study for grades seven, eight, and nine will recognize the suggestions there listed. The chief defect is a failure to list specific objectives. Teachers are enthusiastic for the program of the National Committee, but they do not always see what things to teach to put the new program into practice.

8. The basic list, the subsidiary list and items rated so low as not to be included here, were passed in review before a highly selected list of judges. The mathematics teachers were especially critical. It is recognized that the number of judges is very low. This is a matter of desire and not of convenience. The writer has been in a position during the last few years to obtain hundreds of judgments almost as easily as ten on each item, but he has no faith in the "median judgment of a hundred average judgments." The opinion of one very competent person might easily have expressed finer intuition, insight and vision than numerous mediocre judgments.

9. The technique here employed leaves the door open for *growth.* It is conceivable that this study repeated ten years hence would result in many and drastic changes. Certainly the same study carried out ten years ago would have resulted in a basic list far different from the one here suggested.

It would be possible to get a better initial list by a wider selection. It turns out that indexes and tables of contents of books are not carefully constructed. Obviously another source of improvement lies in an increased number of objective studies.

10. It is not to be expected that this basic list will fit every community, nor win the endorsement of all individuals. On the contrary, the writer does not accept the basic list throughout. In fact, there are a few things in the subsidiary list that he believes far more important than some things included in the basic list. It will be noted that both sets of judges favor some of the items in the subsidiary list very highly indeed. The contention is that this chapter furnishes a good working basis for any group of teachers that sits in conference for the purpose of making a course in mathematics for grades seven, eight and nine for their community.

CHAPTER VI

A REPORT ON SOME STUDIES MADE IN A COOPERATIVE TEACHING PROGRAM

The Refinement of the List of Objectives With Suggestions on Grade Placement.

This chapter will present a number of subsidiary studies supporting the position that materials of instruction can not be selected independent of classroom situations. There is no effort to be complete. It would be impossible to present within the scope of this study even a review of many mimeographed record forms collected by the writer and his colleague in a three year study involving hundreds of classroom teachers and large groups of children. But it is hoped that enough will be accomplished to serve as illustrations. A series of studies will show how a group of teachers may adjust the list of objectives to a community.

Preliminary Teaching Under "Paired" Teacher Plan.

With the guiding principles and the lists of outcomes formulated, the teaching materials were written for each of the three grades. As a preliminary step, these materials were mimeographed and taught to the junior high school classes in the Lincoln High School (New York City). In general, the teaching at the Lincoln School followed the "paired" teacher plan. Very few lessons were taught by the author during the entire period that were not observed by his colleague. No unit of the experimental material was taught without being submitted to the "paired" teacher plan.

After much revision, these materials were printed by the Truan (later the Gazette) Press of Yongers, New York, a small printing establishment which was absolutely independent of any publishing house. This fact is believed to be important as it guaranteed entire freedom to the program to develop as educational rather than as commercial interests dictated.

The Cooperative Teaching Program.

The experimental material was then placed in the hands of more than sixty teachers representing many degrees of teaching ability. In many of the cooperating schools children were classified according to ability as well as they could be with

the tools then available. In this way the materials were tested with the different groups,—Portuguese, Mexican, colored, Polish—in densely populated districts. This seemed absolutely necessary for solving the language difficulties of mathematics. An effort was made to write the material not only *for* the pupil, but *to* the pupil.

The cooperating cities were widely distributed. In some cities only a few classes used the experimental materials, whereas in others a considerable number of pupils participated as may be illustrated by Denver (Colo.), Flint (Mich.), and Fall River (Mass.). In each of these cities approximately one thousand children were kept under test conditions throughout the period of experimentation. This implies that approximately five hundred children in each of these cities studied no materials other than those especially designed for the experiment. The significant details will be presented later in this chapter.

The Four Major Steps in the Program.

At this point it will be helpful to list the four major activities of the experiment, together with a brief comment on each:

(1) The experiment involved the teaching by a teacher of at least two types of instructional material, one of which was constructed especially for experimental purposes; the other was chosen from one of the junior high school texts then available or from some standard arithmetic already in use in the particular school system concerned. Every effort was made to keep the more conventional material entirely within the usual practice of the city. This class was designated the "control" group. It will be obvious that in many cases two or three control classes were available as a check on the experimental class. Some cities neglected the teacher factor and checked the results secured from the experimental class against a large number of classes taught by the other teachers in the system. No data however, secured in this way, are presented in this study.

(2) Emphasis was placed on the critical observation of the children's responses to the types of material taught. Records were kept of pupils' successes, failures, interests, the sufficiency of explanation, the adequacy of practice material, the clearness of terminology, the degree to which the subject matter was taken from the experience of pupils, the extent to which pupils were interested in a topic or a principle even though it did not normally lie within their experience, the

effectiveness of motivation, and the suitability of the projects included. Special blanks* were provided for records in order that the teacher might not find this technique an added problem. No class in any city was accepted as an experimental class from which those conducting the experiment hoped to secure usable records without exacting a promise to keep these records faithfully and report them promptly.

(3) The program made a systematic use of measurement to test the pupils' mastery of each unit of material and growth in skill, information and mathematical power. In this way the order was testing—teaching—testing—and further teaching by means of practice material specially selected to supplement instances of inadequate learning. If a given topic has been taught in November, the only way to discover how much review, if any, is necessary in January, is through a testing program. Due to the expense and labor involved, the tests on individual topics could not be carried through for these topics in all schools.

(4) The judgments of competent critics who were especially interested in perfecting these materials were secured. It is true that a number of teachers were chosen by their supervisors and superintendents to handle these experimental materials because they were known to be poor teachers. This was done to insure against the danger of organizing text materials geared so high as to require a special teacher. The last defensive cry of the hopelessly conservative is: "This program is undoubtedly excellent, but it is twenty years ahead and requires a teacher with special training," forgetting that the traditional materials, if we may judge from the large number of pupil failures, seem to require teachers with very special preparation. On the whole, however, our teachers were gifted and especially selected from the courses in the teaching of mathematics at Teachers College, Columbia University. Most of them had extensive experience and training and were able to make intelligent criticisms including constructive suggestions. Indeed, there is evidence that interest in experimental teaching is itself a highly significant selecting factor. Numerous detailed records were made available from this source, and there is no doubt of their value as will later be illustrated for specific topics.

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*A sample will be given presently.

Drill.

Consider first the matter of drill. There is probably not a more debated issue today among mathematical teachers. Shall we try to give a little exercise to each of many bonds or shall we give much practice on a few things? The reader is probably aware of the practice of text books and teachers. It is not much of an exaggeration to say that the conventional system used by New York City public schools is an effort to have pupils become efficient in adding, subtracting, multiplying, and dividing everything that can be added, substracted, multiplied, and divided. In some cities a ten minute daily drill is required by order from headquarters. What happens at the beginning of each hour? The teacher steps to the blackboard and extemporizes drill materials. She has only faint notions as to (1) the bond (if any) she is strengthening, (2) what pupils (if any) are weak on a given bond, (3) how many pupils already exceed a reasonable standard of skill. And so day after day the few are drilled at the expense of the many.

The National Committee on Mathematical Requirements with the assistance of nearly a hundred mathematical organizations undertook a national campaign to decrease the amount of manipulation of symbols and to increase the amount of purposeful work with verbal problem materials. This program with few exceptions met with hearty approval in scores of mathematics meetings. In fact it is difficult to find anyone who does not approve of the Committee's effort *in theory*,—but practice is another matter. Many teachers traditionally trained and accustomed to many years of teaching courses involving probably eighty percent of meaningless manipulations miss the complicated factoring problems, the "apartment house" fractions, and the "nests" or parentheses, as well as other old friends. They beg for more drill materials when they undertake to carry out the program suggested by the National Committee. What we need is not more drill materials of the traditional type. The data presented in an earlier chapter constitute clear evidence that what we have in the traditional courses does not function to any considerable extent, and it is not likely that we could get appreciable increase of skill by increasing the amount of drill. A physician giving an overdose of the wrong medicine would probably not decide to increase the amount.

Our experiments were constructed in accordance with certain principles from the psychology of drill. It is safe to pre-

dict that these principles will receive wide acceptance during the next decade. These principles may be illustrated by the following:

(1) There should be much practice on a few skills rather than a little practice on each of many things.

(2) A drill exercise must be specific.

(3) Drill, to be effective, must be individual.

(4) Drill must provide a scoring technique so that the pupil may know the degree to which he succeeds.

(5) Drill should make possible the diagnosis of individual difficulties.

(6) Practice should be distributed in decreasing amounts at increasing intervals.

(7) A drill exercise should be standardized so that the pupil may have at least a rough notion of his degree of skill.

(8) A drill exercise should be relatively short. Probably an approximate time for concentrated work in the seventh and eighth grades is between three and five minutes.

(9) A drill exercise should set up a high standard of accuracy. The scheme for drill should make it possible for the pupil to take the same exercise again and again until he has achieved an accuracy that allows only the occasional slip.

The foregoing is probably a list sufficient to illustrate the very definite guiding principles that the authors have had in mind. How effective are these principles in contrast with the usual "hit or miss" plan? Table A is a report of 960 seventh grade children; 477 of these children were in fifteen experimental classes and 483 in fifteen control classes. For every experimental class taught by a teacher there was a control class approximately equal in size and ability taught by the same teacher. The children in the experimental group did one or more of the forty-six goals* of arithmetic for four minutes each day at the beginning of the recitation period. The children in each control group did the work as presented in the conventional text book which at the time of the experiment was the adopted text book of the particular city concerned.

*These goals are now distributed throughout the "Mathematics for the Seventh School Year" and "Mathematics for the Eighth School Year." Schorling and Clark, World Book Company, Yonkers, New York.

A considerable number of different texts extensively used were studied by the control groups. The teachers were asked by the

TABLE A

Showing Achievement of 960 Seventh Grade Pupils at End of School Year

School	Number of Pupils		Score on Woody-McCall	Excess in Favor of Experimental	Teacher's Judgment
Winfield, Kan.	Experimental	50	22.6	+2.9	Experimental
	Control 52		19.7		
Belleville, Ill.	Experimental	35	26.5	−0.9	
	Control 37		27.4		Control
Denver, Colo. Class A	Experimental	38	26.0	+1.7	Experimental
	Control 35		24.3		
Denver, Colo. Class B	Experimental	38	26.3	+2.3	About equal on Terman group
	Control 39		24.0		
Denver, Colo. Class C	Experimental	37	21.0	−0.7	About equal Terman group
	Control 32		21.7		
Denver, Colo. Class D	Experimental	34	23.5	+1.0	
	Control 33		22.5		Control
Flint, Mich. Class A	Experimental	23	21.5	+1.5	Equal
	Control 20		20.0		
Flint, Mich. Class B	Experimental	33	24.8	+1.3	Equal
	Control 33		23.5		
Detroit, Mich. Class A	Experimental	26	25.4	+2.5	Experimental
	Control 33		22.9		
Detroit, Mich. Class B	Experimental	33	22.9	+0.8	Experimental
	Control 33		22.1		
Detroit, Mich. Class C	Experimental 33		23.6	+2.2	
	Control 27		21.4		
Boonton, N. J.	Experimental	20	26.4	+0.1	
	Control 30		26.3		Control
Tulsa, Okla.	Experimental	22	24.5	+2.0	About equal
	Control 24		22.5		
Toledo, Ohio Class A	Experimental	28	25.1	+0.8	Experimental
	Control 28		24.3		
Toledo, Ohio Class B	Experimental	27	26.8	+0.0	Experimental
	Control 27		26.8		

experimentors to use every known device (achievement tests, intelligence tests, teacher judgment, etc.) to make the two sections equal in ability.

Table B presents a similar study for 1040 eighth grade children; 533 of these children were in nineteen experimental classes whereas 507 were in the nineteen corresponding control groups. The columns of Table B present the same facts as the corresponding columns in Table A.

It will be noted that the second column of the table indicates the number of pupils enrolled in the various sections. The

TABLE B Showing Achievement of 1040 Eighth Grade Pupils at End of School Year

School	Number of Pupils		Score on Woody-McCall	Excess in Favor of Experimental	Teacher's Judgment
Omaha, Neb.	Experimental 20 Control 18		28.2 28.0	+0.2	About equa
Boonton, N. J.	Experimental 20 Control 15		30.5 28.5	+2.0	Experimental
Medina, N. Y. Class A	Experimental 13 Control 14		28.0 27.0	+1.0	About equal
Medina, N. Y. Class B	Experimental 18 Control 19		26.3 25.1	+1.2	Experimental
E. Straudburg, Pa. Class A	Experimental 30 Control 31		26.8 25.6	+1.2	Control
E. Straudburg Pa. Class B	Experimental 33 Control 31		27.5 26.8	+0.7	Experimental
Toledo, Ohio Class A	Experimental 36 Control 38		28.0 28.6	−0.6	Control
Toledo, Ohio Class B	Experimental 40 Control 40		27.0 27.0	+0.0	Control
Flint, Mich. Class C	Experimental 20 Control 20		25.0 24.1	+0.9	Equal
Flint, Mich. Class A	Experimental 22 Control 21		24.5 23.1	+1.4	Equal
Flint, Mich. Class B	Experimental 20 Control 20		23.4 23.4	0.0+	Equal
Flint, Mich. Class D	Experimental 30 Control 25		25.8 25.0	+0.8	Equal
Flint, Mich. Class E	Experimental 25 Control 24		25.0 25.3	−0.3	Equal
Winfield, Kan.	Experimental 41 Control 34		23.3 25.0	−1.7	Control
Belleville, Ill.	Experimental 36 Control 33		28.5 28.5	+0.0	Control
Denver, Colo. Class A	Experimental 33 Control 32		27.7 27.6	+0.1	About equal Terman group
Denver, Colo. Class B	Experimental 32 Control 37		25.4 25.7	−0.3	About equal Terman group
Denver, Colo. Class C	Experimental 36 Control 28		29.8 29.8	+0.0	About equal
Denver, Colo. Class D	Experimental 28 Control 29		26.6 26.1	+0.5	Experimental

third column indicates the score on the Woody-McCall Mixed Fundamentals, Form III. The fourth column indicates the

excess of the median of the experimental group over the median of the control group. When the control group is the higher this excess is indicated by a negative number. The last column shows which section (if any) the teacher believed to be the better section in computation at the beginning of the year. In spite of efforts to equalize sections, the teacher usually has a common sense opinion as to the effectiveness of these balancing devices. It seemed to the experimentor that this teacher judgment might be helpful to the reader.

FINDINGS:

1.—From the table for the seventh grade it will appear that one experimental class equalled in achievement the corresponding control class. Two experimental classes did not do as well as the control classes and twelve of the experimental classes exceeded the corresponding control groups.

2.—Four of the experimental classes of the eighth did as well as the corresponding control groups, four classes did not do as well and eleven experimental classes exceeded the corresponding control groups.

3.—Considering the seventh and eighth grade children as a single experiment, we note that there were altogether thirty-four experimental groups plus thirty-four control groups. Of these thirty-four experimental groups all but six classes either did as well as or exceeded the corresponding control groups.

4.—It must appear obvious that this experiment represents an effort to include a great variety of learning situations (as for example, pupils in homogeneous groups but great differences between groups, teachers with wide range of experience, differences in types of community, and the like.)

5.—The important question is: How significant is this excess in favor of the experimental groups? If all the children in the different experimental groups in these various cities were collected into one large room and if all the children in the different control groups were collected into a single room and if twenty pupils were rejected at random from the experimental class groups so as to make the number remaining in the experimental group exactly equal to the control group, then it is almost certain that the experimental group would, if it maintained the speed and accuracy revealed in our test results, turn out from five thousand to six thousand correct problems more in a single hour than the pupils of the control group. This would seem to be a very significant difference of product.

6.—While the Woody-McCall Test is a twenty-minute test, the fact is that the productive time is approximately from eight to twelve minutes. That is to say very little is accomplished by most pupils after the first half of the period. With this in mind it is a reasonable guess that the seventh grade children in the experimental group alone could do correctly about five thousand problems more than an equal number of children in the control group. The difference in product for the eighth grade is obviously not so significant as the difference in achievement between the seventh grade pupils.

THE USE OF RECORD FORMS ON UNITS OF INSTRUCTION

The cooperating teachers were asked to fill in a record form on each unit. A sample form sent out on the first unit for the second experimental edition of the seventh grade is given here. The responses written in by the cooperating teacher are underlined. For a full interpretation it would be necessary to inspect the twenty-one pages on which this record form is based. It will probably meet the reader's needs if we list the activities of the twenty-one page unit as follows:

1. Making careful measurements with the ruler.

2. A discussion, with illustrations, of how accurately people measure, and a discussion of practical difficulty of precise measurements.

3. The reading of line segments drawn on squared paper (as a basis for scale drawing and graphic work), and measuring with the metric system. In the discussion of the metric system we had nothing more in mind than to give the pupils a "feeling for" the length of a millimeter, centimeter, and meter. This last unit had two purposes, one of which was to motivate work with decimal fractions.

Here follows the record form:

RECORD FOR MATHEMATICS FOR THE SEVENTH SCHOOL YEAR—CHAPTER I.

Name of School: *Byers Junior High.*
City: *Denver*
Teacher: *Ella C. Johnson*
Grade: *7b*
Number of pupils in experimental group: *40* Number of pupils in control (regular) group: *40* Length of recitation

period: *55* Number of recitations in mathematics per week in your school: *5* Do you give homework to your pupils? *No* If so, how many minutes per week? On what basis were your pupils classified?

Teacher's marks

Thorndike-McCall Reading Test
Buckingham Reasoning Test
Denver Spelling Test
Briggs English Test

1. How many lessons were required to teach Chapter I.? *18*

2. Underscore the right word: I found this chapter (very difficult, difficult, *easy,* very easy) to teach.

3. What section or what pages do you regard as too difficult for your children? Answer: *Individuals had difficulty but majority of group no trouble.*

4. What per cent of your pupils found this chapter
   very interesting *27½%*
   interesting *70%*
   a task to be done *0*
   having no appeal whatever *2½%*

5. List here the sections in Chapter I that you would omit, together with a brief reason for each. *So far as my group is concerned, no omission. Various groups react differently, and I would feel it necessary to handle a different group before making suggestions.*

6. Shall the word "segment" be omitted? *No*

7. Is section 7 any help at all in motivating the use of the compasses? *Yes*

8. Please vote Yes or No on this: Do you want to teach the metric system in the seventh grade? *Depends on the group a good group, yes.*

9. Were your pupils able to do Ex. 2, page 13? *Yes*

10. Were your pupils interested in Ex. 8, page 16? *Yes*

11. What suggestions have you for improving section 13? *Not enough practice in finding length of line to second decimal place.*

12. Did your pupils add any interesting material to 14? *Grinding of cylinders must be accurate to one part in one million, otherwise oil leaks out.*

13. Were Exs. 1, 2, and 3 too difficult? (Keep in mind that we are not trying to teach all about per cent in this chapter. It is a mere touch of things to come.) *No.*

14. Get pupils started on tests 1 and 2. Let each one work at his own rate on these tests throughout the book.

15. List here your general impressions of this unit.

Since linear measure is used in stores, on farms, in building, in athletic games, etc., the pupils might feel that it was useless to learn a system which would seem to have so little connection with every day life. I would suggest that the occupations using the metric system be given as well as the way in which it is used.

For slower groups I believe more practice in per cent should be given though it is to be met later. This chapter is a foundation for following work and should therefore give the child something on which to build.

Raleigh Schorling
John R. Clark

The Lincoln School, 425 West 123rd Street,
New York City.
October 3rd, 1922.

Findings: Uses of Teachers' Records. These records were sent back by 42 cooperating teachers in sufficiently good form to be of use. We are now ready to consider some of the questions on which these records throw some light. These are:

1. How much time did the teachers give to this 21-page unit?
2. How much home work did these teachers require?
3. What sections of the unit were found to be too difficult?
4. Is the metric system hard to teach?
5. What topics do these teachers wish to omit?
6. To what extent did pupils find this unit interesting?
7. How many suggestions did the 42 teachers make in response to "Did your pupils add interesting material to Section 14" and what were the significant suggestions?

Consider first the time required. The following table gives the number of recitation minutes which the various teachers devoted to this unit:

TIME REQUIRED IN CLASS ROOM

NUMBER OF RECITATION MINUTES REQUIRED TO TEACH THE UNIT

Teacher Number	No. of Minutes	Teacher Number	No. of Minutes	Teacher Number	No. of Minutes
1	1100	15	1125	29	750
2	990	16	540	30	400
3	755	17	420	31	450
4	750	18	900	32	1650
5	450	19	630	33	1125
6	1045	20	1800	34	900
7	1210	21	630	35	800
8	1595	22	910	36	550
9	400	23	780	37	900
10	1350	24	600	38	255
11	540	25	768	39	945
12	510	26	550	40	720
13	990	27	1120	41	810
14	900	28	540	42	540

Findings: It is astonishing to note that the range is from 250 minutes to 1,850. Even if we cut off the two extremes, the range is still enormous. The average seems to be approximately 808 teaching minutes, which is about 13½ recitation periods with 60 minutes "in the clear." The enormous range suggests that certainly teachers are entitled to friendly guidance, this guidance to be given in the form of a suggestion as to about how long it should take to teach a unit adequately. There is no intention here of fixing a definite limit but certainly it would be helpful to the teacher—the beginning teacher—if she were told that in general it should take from 10 to 15 recitation periods to cover a given unit.

How much home work did the teachers require per week? The amount as judged by the teachers is surprisingly small, as is shown by the following table:

MINUTES OF HOME WORK PER WEEK

No. of Teachers	No. of Minutes	No. of Teachers	No. of Minutes
19	0	1	80
1	20	1	90
2	30	1	100
3	40	2	120
2	50	1	150
5	60		

Of the 38 teachers whose records were clear on the matter, 19 gave no home work whatsoever, and even the largest amount —150 minutes, does not appear excessive. It is probable however, that the amount was greatly underestimated by teachers. The general situation, then, as judged by the teach-

ers was that half the teachers gave no home work and that the other half gave approximately an averge of one hour per week.

What sections, topics or problems of this teaching unit were found to be too difficult? Sixteen teachers said that no part of Chapter I was too difficult for seventh grade children. There were, however, 11 suggestions. These are found in the following table:

No. of Teachers	Things in Measurement Found too Difficult	No. of Teachers	Things in Measurement Found too Difficult
1	Page* 10, Ex. 15	1	Pages 16 and 22
3	Page 23, Ex. 2	2	Page 18, Sec. 3
16	Page 17, Sec. 13	2	Page 19
1	Last Part Sec. 8	1	Page 11, No. 16
1	Section 14	1	Page 10
		1	Page 13

Is there any question as to which topic, if any, in this unit needed adjustment? Sixteen teachers chose page 17, Section 13, as too difficult for seventh grade children.* This particular topic will serve as an excellent illustration of the method of adjusting a unit of subject matter to learning difficulties encountered. This unit attempted to give pupils ability in reading the lengths of line segments as used in the scale drawing or in the graph. The effort was made to have pupils read to the second decimal place. From the teachers' responses it was decided to put in a preparatory study in the form of a page with two illustrations using graph paper to teach pupils to read to the tenth of an inch; in short, an intermediate learning step was designed. The responses on the teachers' records from the third edition indicate clearly that this particular topic no longer presents especial difficulty.

Is the metric system hard to teach? This question is of some interest because many junior high school teachers have not had special preparation in college courses in mathematics or they were prepared in the days when the metric system was not taught. The following table includes the responses on this question:

*Referring now to 2nd Experimental Edition of Mathematics for the Seventh School Year—Schorling-Clark; filed at the Bureau of Reference and Research, University of Michigan.

Degrees of Difficulty	*No. of Teachers*	*Per cent*
Very difficult	2	—5
Difficult	16	+38
Easy	22	+52
Very easy	2	—5

Of the 42 teachers responding, it appears that 24 found it easy or very easy to teach. Considering some of the factors involved, which have already been enumerated, the responses are satisfying.

It is interesting to follow up the preceding question with a related question: How many teachers want to teach the metric sysem? The vote here was: 36 responded "Yes" and 6 "No." There was clear evidence that the teachers found the metric system a great help in teaching decimals.

What topics do the teachers wish to omit? The following table is of interest because here the teachers struck at details, as for example, problems rather than topics; neither were the things that the teachers wished to omit the same that they found difficult. (This is not evident from a comparison of our tables. The point is that the teacher who found a section difficult is not necessarily the same teacher who wished to omit that section.)

Twenty-four teachers did not choose to omit any part of this unit. Nine wished to omit section 13, discussed under a previous question, and a considerable number wished to omit page 11, problems 15, 16 and 17, with a few problems here and there.

THINGS TO BE OMITTED

No. of Teachers	To be Omitted	No. of Teachers	To be Omitted
5	Page 11, problems 15-16-17	2	Page 7
9	Section 13, Page 17	2	Page 23, Ex. 2
2	Last part of Section 8	1	Page 22, Ex. 8
2	Section 7	1	Section 3
1	Ex. 3, Page 18	2	3 problems, Page 19

Problem 16 read as follows: "A mile equals 5,280 feet. If an army advances one kilometer (km.) (A kilometer equals 1,000 meters), how many miles has it moved?" From the point of view of the classroom this was a poor problem. Needless to say, it was omitted in the next experimental edition of the teaching materials.

To what extent did pupils find this interesting? The following table shows the responses of 1,195 pupils.*

TABLE SHOWING PUPILS' INTERESTS

Degrees of Interest	No. of Pupils	Per Cent
Very interesting	532	44
Interesting	494	41
A task to be done	144	12
Having no appeal whatever	25	2

If we add the first two groups, it appears that 85 per cent of the pupils found this unit challenging, as judged by their teachers. Of special interest is the group who found this unit a "task to be done." Fifty-six of these were found in three classes, as follows: 32 (out of 53); 18 (out of 37); 6 (in a class of 41). It is significant to note the size of these three classes in which the largest groups of uninterested pupils were found.

How Significant Were the Suggestions Made by These Forty-Two Teachers in Response to "Did your Pupils Add Interesting Material to Section 14?"

The topic of section 14 was: "How Accurately People Measure." Statements about fine measurement made by machinists, surveyors and chemists were given.

To the question in the record form, seven teachers made no response, eight said 'yes' without further comment and five replied 'no.' From the remaining teachers the following replies were received:

> Grinding of cylinders must be accurate to one part in one million, otherwise the oil leaks out.—Byers Junior High School, Denver, Colorado.
>
> One boy heated a metal rule, then measured it.—Dort Junior High School, Flint, Michigan.
>
> Brought up this question: "How accurately does the city sealer of weights and measures make his tests?—Homedale School, Flint, Michigan.
>
> Scales at the mint near here were discussed, also surveying—Morrey Junior High School, Denver, Colo.

*These records include classes in Flint (Michigan); Denver; Bellville (Illinois); Duluth; Winfield (Kansas); Detroit; Fall River; Touson (Md.); Boonton (N. J); East Moriches (N. Y.); Medina (N. Y.); Oxford (O.); Yellow Springs (O.); Toledo; Swarthmore (Penna.); Richmond (Va.); Winchester (Va.); Greenwich (Conn.); Helena (Ark.); Winnetka North Shore County Day School (Illinois).

Several illustrations were given, such as surveyor's measures and the use of the compass and the sun dial. —Dewey School, Flint, Michigan.

The children related incidents that were personal to them and proved helpful in accuracy.—Oak Street School, Flint, Michigan.

The pupils had an interesting discussion of the work of mechanical engineers.—The Walker School, Flint, Mich.

They discussed why some balances are enclosed in glass.—Stevenson School, Flint, Michigan.

Many specific examples were mentioned which required very accurate measurements.—Junior High School, Winfield, Kans.

A boy told how people measure ball bearings.—Hutchins Intermediate, Detroit, Michigan.

We had very interesting themes on the accuracy of machinists.—Henry Lord School, Fall River, Mass.

A boy discussed the difficulty encountered in some of our factories in changing from metric system to our English system.—Navarre School, Toledo, Ohio.

The topic opened up a new line of thought for my pupils.—The Watson School, Fall River, Mass.

The fathers were interested and contributed excellent suggestions to children. The children not only see things when they come in contact with them, but they are now looking for them.—Lathrop Avenue School, Boonton, New Jersey.

Some scales are so accurate and delicate that a hair can be weighed.—Junior High School, East Moriches, N. Y.

Pupils spoke of fine screws which machinsts use for measuring.—Medina School, Medina, New York.

One of the children is a son of a professor in science and he had interesting material.—Yellow Springs, Ohio.

The day after the lesson several children were anxious to tell how accurately their fathers measured in their line of work, as for example, printers. They gave a few instances in which errors would have serious results.—East Side Central, Toledo, Ohio.

One pupil was related to an electrical engineer and another to a chemist. Both added material.—Swarthmore, Penna.

The domestic science girls discussed measurements in cooking and sewing. The boys gave illustrations from their wood-working course.—Winchester High School, Winchester, Virginia.

One boy's father is a surveyor and he told the class a little about his father's work.—Helena High School, Helena, Kansas.

A careful reading of the preceding twenty-one suggestions will probably convince the reader that these materials would be of great assistance to the people who are writing materials for children. The record form used by the experimentors suggests to supervisors and school executives a method for humanizing their courses.

Refinement of Materials and Grade Placement.

This concludes our illustrations of the method used by the experimentors in their three-year program of refinement of objectives. The many interesting and helpful questions that grew out of the use of the single record form given in this chapter furnishes evidence why the two experimentors were willing to continue this tedious and detailed process for three years with the seventh grade materials and two years with the eighth grade materials. Approximately eight hundred record forms, similar to the one used in this chapter, were collected in the year 1922-23 alone. Not only was convincing evidence secured by this technique on such major questions as: (1) The grade placement of a particular problem, principle or topic; (2) The choice of geometric materials, of algebra materials, etc., (3) The relative difficulty of geometry and algebra; and (4) The proper distribution of materials and recall, but also it was the only method known to the author whereby a given bit of subject matter can be placed "under the microscope" in an actual teaching situation. Numerous illustrations including detailed test results could be given at this point if space permitted.

Recently a noted research scholar in the field of history was describing his method of constructing a textbook for the secondary school. He said: "Our text was first taught in mimeo-

graphed form by a very wise high school teacher. She wrote comments on the margin. It is my guess that she improved the manuscript something like fifty percent."

This chapter has attempted to describe a technique that can be and must be used to do in systematic fashion what the learned scholar found valuable.

BIBLIOGRAPHY

Adams, H. W.—*Unpublished Master's Thesis.* Department of Education, University of Chicago.

Bobbitt, Franklin—*Curriculum Making in Los Angeles.* Article in Elementary School Journal, October, 1924.

Bonser, Frederick—*Elementary School Curriculum.*

Briggs, T. H.—*The Junior High School*, p. 157.

Caldwell, Otis W.—Introduction to "Mathematics for the Eighth School Year"—Schorling-Clark.

Camerer, Alice—*What Should Be the Minimum Information About Banking?* Seventeenth Yearbook of the National Society for the Study of Education, Part I, Pp. 18-26.

Charters, W. W.—*Curriculum Construction*, Pp. 231-236.

Chase, Sara E.—*Waste in Arithmetic*, Teachers College Record, Vol. 18, September, 1917.

Clark, J. R., Schorling, Raleigh, and Rugg, Harold—*Inventory Test.* Gazette Press, Yonkers, New York.

Commission on the Reorganization of Secondary Education—Cardinal Principles of Secondary Education.

Dewey, John—*Elementary School Teacher*, 4:193-204; *The International Journal of Education*, 26:315; *Human Nature and Conduct*, p. 24; *Interest and Effort*, p. 35.

Gallaway, Mrs. T. T.—Unpublished study reported in Charters' "Curriculum Construction." Pp. 241-243.

Guiler, Walter—*Doctor's Dissertation*, filed at University of Chicago, Chapter on List of Objectives Based on Courses of Study.

James, William—*Talks to Teachers*, 66f.

Jessup, W. A., Coffman, L. D.—*The Supervision of Arithmetic*, Chapters I and II, Pp. 1-38.

Meriam, Junius—*Child Life and The Curriculum.* Chapters 13 to 16.

Mitchell, H. Edwin—*Some Social Demands on the Course of Study in Arithmetic.* Seventeenth Yearbook of the National Society for the Study of Education, Part I.

Moore, Ernest C.—*Minimum Course of Study*, Chapter I, Pp. 1-41.

National Committee on Mathematical Requirement—*The Reorganization of Mathematics in Secondary Education*, Pp. 117-278; 45 and 46; 57-59; Chapter XII.

Nunn, T. Percy—*The Teaching of Algebra*, 1914. Pp. 78, also Pp. 14, 15.

Rugg, Harold and Clark, J. R.—*Reorganization of Ninth Grade Mathematics.*

Schorling, Raleigh—*Experimental Courses in Mathematics:* Reorganization of Mathematics in Secondary Schools, Chapter XII.

Schorling, Raleigh and Clark, J. R.—*Mathematics for the Seventh School Year.* Second Experimental Edition, filed at the Bureau of Reference and Research, The University of Michigan.

Schorling, Raleigh and Clark, J. R.—*The Present Status of Mathematics in Grades Seven, Eight, and Nine.* The Mathematics Teacher: Vol. XIV, Pp. 264.

Schorling, Raleigh and Clark, J. R.—*Mathematics for the Seventh School Year.* World Book Company, Yonkers, New York. Chapter I, and Instructional Tests.

Schorling, Raleigh and Clark, J. R.—*Mathematics for the Eighth School Year.* See the Introduction and Instructional Tests.

Thorndike, E. L.—*The Psychology of Arithmetic.* Pp. 103; *The Psychology of Algebra.* Chapter XII, also p. 70.

Wilson, G. M.—*The Social and Business Uses of Arithmetic,* Pp. 1-62, *Connersville Course in Mathematics,* Warwick & York, Baltimore, Maryland. Pp. 6-10.

Wilson, G. M. and Hoke—*How to Measure.*

Wise, Carl T.—Elementary School Journal: Vol. 20, No. 2, October, 1919, Pp. 118-136.

Woody, Clifford—Article in Elementary School Journal, March, 1922, Pp. 505-521.

Woody, Clifford—*The Use of Calculating Devices in Business and Their Influence on the Use of Arithmetic.* Third Yearbook, Department of Superintendence, 1925. Contribution to Arithmetic Section in Third Yearbook, Department of Superintendence.

Woody, Clifford and McCall, William T.—*Test Form III.* Teachers College Bureau of Publications.

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